Using machine learning to increase efficiency in design of experiments for cyclic characterization of fibre-reinforced plastics

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Abstract

Efficient characterization of fatigue behavior plays a crucial role in engineering design as it reduces the financial costs associated with expensive experimental tests. Existing methods for characterizing the fatigue behavior of fibre-reinforced plastics have proven inefficient due to the oversight of important design parameters, such as fibre orientations. To address this challenge, we propose an innovative approach based on Gaussian process regression. Our approach integrates previously unaccounted design parameters into the decision-making process, ensuring that optimal design points are selected for testing. By doing so, we maximize the gain of knowledge within the model, resulting in improved efficiency and accurate characterization of fatigue behavior.

Keywords

fatigue behavior, machine learning, gaussian processes, fibre-reinforced plastics

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1. Motivation

The significance of lightweight design and efficient material usage in different economic sectors is emphasized by increasing demands resulting from upcoming leading initiatives and legal requirements related to climate targets and the transition to a more resource-efficient economy. In recent years, lightweight materials like fibre-reinforced plastics (FRPs) have gained greater importance in new industries, including the automotive industry, due to their excellent specific stiffness and strength. While these materials have been well-established in other industries such as wind energy and aerospace for many years, designing components to perform optimally under both static and fatigue loads remains a significant challenge [1].

While FRPs offer significant potential for lightweight applications, they also exhibit highly complex fatigue behavior. Developing reliable fatigue models can play a pivotal role in advancing lightweight design by reducing safety factors. However, characterizing the fatigue behavior of FRPs requires a substantial number of experiments, primarily due to the multitude of influential design parameters, particularly fibre orientations. Consequently, the complete characterization of the fatigue behavior of FRPs is a time-consuming and costly process. This can be significantly reduced by implementing a good design of experiments (DOE). Compared to current DOE a more efficient approach would at the same time yield better insights into the fatigue behavior by only requiring the same number of experiments and achieve equivalent knowledge with fewer experiments.

2. State of the Art

The basis for understanding the fatigue behavior of FRPs are experimentally determined S-N curves, sometimes called WÖHLER curves. S-N curves are determined by subjecting test specimens to cyclic loads at constant amplitudes until failure. A S-N curve is divided into the regions of low cycle, high cycle and long life fatigue as shown in Figure 1. The anisotropic material behavior of FRPs requires the determination of multiple S-N curves at different fibre orientations to accurately predict fatigue life and failure mechanisms in FRP structures. By testing specimens at various fibre orientations and interpolating between the curves the tolerable number of load cycles for a specific combination of stress level and fibre orientation can be estimated.

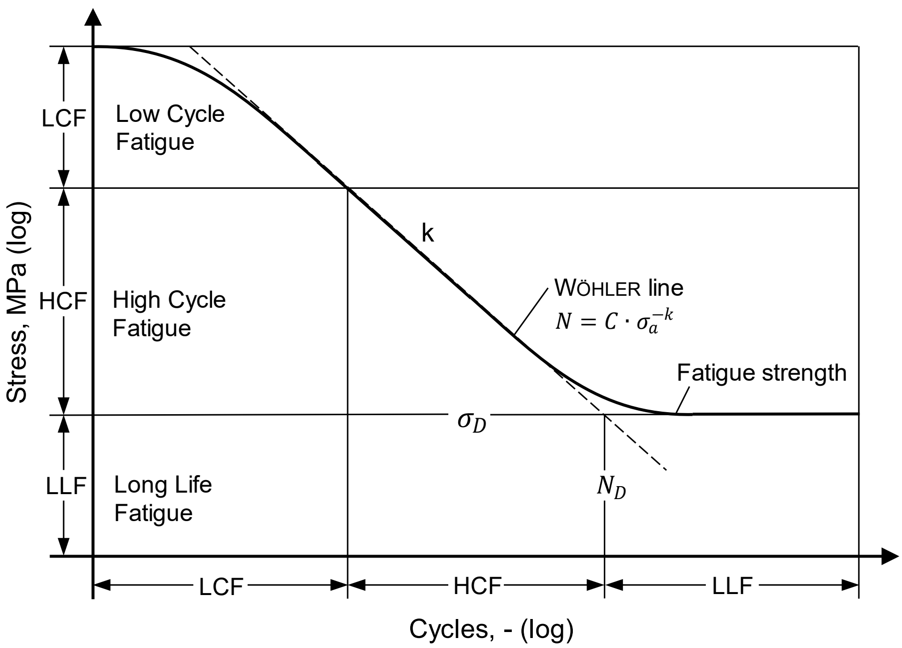

Figure 1: Values of a S-N curve and classification of areas according to [2].

The experimental determination of S-N curves typically relies on standardized methods, such as the German standard DIN 50100, which defines two commonly used test strategies: the horizon method and the pearl-string method [3]. The horizon method involves conducting several fatigue tests at selected stress horizons (Figure 2a). It is important to select the stress horizons exclusively within the high cycle fatigue region and as close as possible to the transition areas for low cycle and long life fatigue. However, this requirement assumes prior knowledge of the approximate positions of the three regions, which highlights a disadvantage of the horizon method [4].

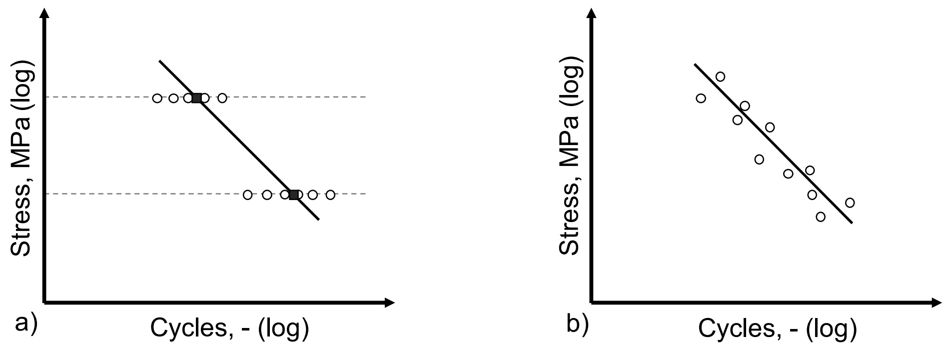

Figure 2: Methods of fatigue testing in the finite life regime: 2a) Method of fixed load horizons, 2b) Method of discrete load steps.

The determination of S-N curves can also be achieved using the pearl-string method as described in the German standard DIN 50100 [3]. This method is well suited if the position of the long life fatigue area cannot be estimated before the tests. The fatigue tests are performed at many different load levels within the high cycle fatigue regime and are arranged along a line as on a string of pearls (Figure 2b). First, a loading level within the high cycle fatigue regime (10^4 to 10^6 stress cycles) is determined. The stress level is then gradually adjusted to approach the transition ranges to low cycle and long life fatigue [3, 5].

The pearl-string method is considered an adaptive sampling method in fatigue testing. Adaptive sampling methods adjust the selection criteria throughout the experiment based on preliminary results as they come in [6, 7]. Adaptive sampling refers to the practice of dynamically modifying the sampling strategy during an experiment to optimize the collection of data. This approach allows for the incorporation of new information and the refinement of the sampling process as the experiment progresses. However, the adaptive pearl-string method as described in DIN 50100 has two issues. Based on the incoming results the method only varies the applied load, while all other design parameters remain fixed. Moreover, it gives only a general direction in which the load should be changed, but not a specific method of calculation or value.

3. Research problem and research goal

The previous sections have shown that characterizing the fatigue behavior of FRPs is a challenging task. The existing methods for determining S-N curves create the first problem. Whereas the horizon method requires you to know the positions of the fatigue regions before you start testing, the adaptive pearl-string method only provides a rough idea of where to put the next design point. This leads to inefficiency in an area where time and testing capabilities are limited. In addition, both methods fail to capture that the fatigue behavior of FRPs is influenced not only by the magnitude of the applied load – but also by the orientation of the fibres relative to the direction of force, the frequency of cyclic loading and environmental conditions such as temperature and humidity. Together, these influencing factors span a wide parameter space that current methods cannot capture.

Consequently, these two points lead to an inefficient and incomplete characterization process of the complex fatigue behavior of FRPs. The challenges encountered in characterizing the fatigue behavior highlight the need for more efficient methods that can comprehensively account for the various influencing factors. This leads to the central research question of this study: How can the fatigue behavior of FRPs be characterized more efficiently? Specifically, the study aims to develop a methodology that incorporates all of the multiple influencing factors, including the intricate variations in fibre orientation. Machine learning techniques, in particular Gaussian process regression, will be used to achieve this. The following sections address this research question in detail.

4. Methods and procedures

4.1. Overview

The basic procedure of the new method is shown in Figure 3. It is based upon the adaptive sampling method proposed in [8], but modified for the usage with FRPs. This new type of DOE – in this respect similar to the pearl-string method – adaptively selects the next design point based on the experiments already performed [3, 5]. However, it can also incorporate parameters that were previously unconsidered in the selection process. It is built upon a Gaussian process regression model, a type of supervised learning model that provides a built-in estimate of uncertainty. The concept uses the predicted uncertainties combined with a utility function that favors the exploration of areas with high uncertainties. As it can make the most of the available data, it is particularly suitable for dealing with small data sets. While leading to an optimal usage of the limited resources, it will also produce a better understanding of the complex fatigue behavior of FRPs [9]. Each point's uncertainty is visualized by a colormap. Areas with a high uncertainty are marked with a reddish color, whereas areas with bluish colors show that the model is relatively certain of its prediction. After each experiment, these uncertainty calculations suggest a new design point, until the test is finally terminated after a defined number of conducted experiments. In this way, those test points are tested that provide the greatest gain in knowledge to the model.

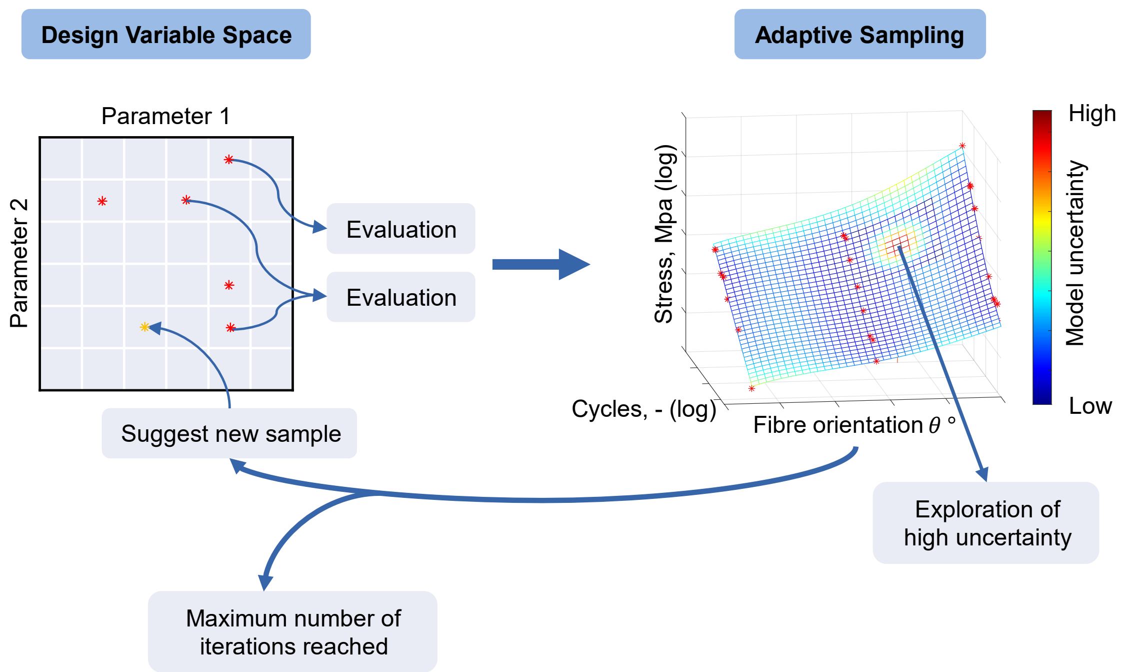

Figure 3: Usage of Gaussian process regression for increasing efficiency in design of experiments (based on [8]).

4.2. Gaussian process regression

Gaussian processes (GPs) are a versatile class of models used to describe functions. In basic terms, a GP is a way of representing functions in a probabilistic way. The power of GPs is that when we consider a finite set of function values, such as $f(x_1)$, $f(x_2)$,…$f(x_n)$ their joint distribution follows a Gaussian distribution [10]. To fully define a GP model, two key components are needed: the mean function and the covariance function, also known as a "kernel" in Gaussian process regression. The mean function represents the average behavior of the function we are modelling. It is common to assume a mean of zero everywhere, as any uncertainty about the mean can be incorporated into the kernel. Once we have taken the mean into account, it becomes the kernel which determines how the GP model behaves. It plays a crucial role in capturing the patterns and relationships in the data. It defines how the model generalizes or makes predictions for new, unseen data points [11]. Figure 4 depicts a GP modeling a one-dimensional function. The left plot illustrates the GP conditioned on three data points, showing the predictive mean and 95% confidence intervals. These confidence intervals indicate the range within which the true function value is likely to fall with 95% certainty. In the right plot, the GP is conditioned on four data points, resulting in smaller confidence intervals. This highlights how more data points lead to tighter confidence intervals in GP modeling.

The exciting aspect about GPs is that there is a wide range of choices for the covariance function. Just by choosing different kernels, we can specify different models. GPs have found wide application in various fields due to their versatility and effectiveness, including geostatistics. In geostatistics, one of the practical uses of GPs is a technique called kriging, which helps to estimate values at unobserved locations based on observed data by considering the spatial variation and correlation captured by GPs. In simpler terms, kriging uses GPs to understand how nearby locations relate to each other, allowing us to predict values in places where we haven't collected data. One challenge in using GPs is to find the right kernel that accurately represents the structure present in the data that we want to model. This task involves constructing a kernel that captures the specific patterns and relationships that exist. In the next part we will take a closer look at kernels and how they can be used to represent the structures observed in fatigue testing.

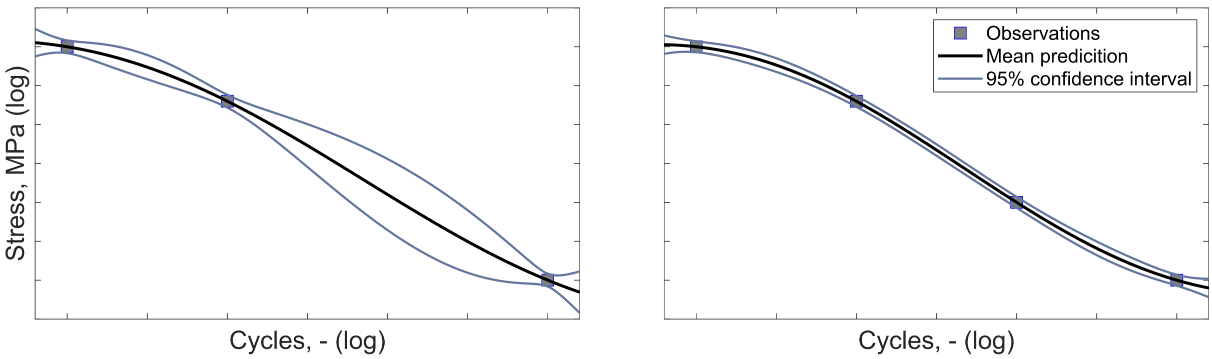

Figure 4: A visual representation of a Gaussian process modeling a one-dimensional function. Both plots have the same axes.

4.3. Covariance function

A covariance function is a mathematical function that quantifies the relationship between pairs of input data points in Gaussian process regression. It determines how the output values of the regression model vary based on the similarity or dissimilarity of the input points. The covariance function is crucial for estimating the uncertainty and smoothness of predictions in Gaussian process regression.

In the case of experimental fatigue testing, where only a few data points are available due to time and cost constraints, it is crucial to choose modeling approaches for the covariance functions that incorporate existing physical and engineering knowledge. The covariance functions must map factors such as symmetry, periodicity and monotonic behavior [12]. By using specific kernels, we can intuitively describe individual phenomena and easily combine them together. A very advanced software solution used for this purpose in this work is GPflow [13]. For example, the relationship between the applied load and the maximum number of tolerable cycles can be described by a linear kernel. On the other hand, the nonlinear influence of the fibre orientation can be accurately represented by using a periodic kernel. Sometimes it also necessary to combine individual kernels to create new ones. This allows influencing factors such as stress ratio, frequency, temperature, and humidity to be considered in the model.

4.4. Sampling Constraints

When determining the next design point that provides the greatest gain in knowledge to the model, the acquisition function has to deal with physical constraints and real word limitations. To perform fatigue tests, specimens of the material to be tested must be prepared with the desired fibre orientation. Since each additional fibre orientation is associated with additional costs and time delays, S-N curves are usually determined only for the fibre orientations 0°, 45° and 90°. If only specimens with these orientations are available, the selection function must take this into account when choosing the next design point. However, it is also conceivable that samples with one more orientation can be produced. In this case, the acquisition function should determine the fibre orientation with the greatest possible gain in knowledge – while considering manufacturing-related limitations when making the choice.

It has been shown that a temperature increase of more than 10°C can lead to a rapid decrease in strength and undesired thermal failure [14, 15]. An excessive temperature rise can be caused by internal friction during testing. This is why often a maximum frequency limit is imposed. Therefore, the acquisition function must select a frequency within the allowable temperature rise limits.

Another constraint to address is the selection of the applied load level. It is important to note that force measurements inherently involve uncertainty. Therefore, when determining the next design point, the function must consider this uncertainty and make decisions accordingly. By acknowledging and incorporating these physical constraints and real-world limitations, the GP-based model can effectively guide the selection of design points, ensuring practicality, accuracy, and relevance in the fatigue testing of FRPs.

5. Results and discussion

In the following, the GP-based method is compared to the pearl-string method through a benchmark. The necessary fatigue tests are conducted virtually, allowing for a significantly larger number of measurements. This approach enhances the meaningfulness of the results, as failure tests always entail a certain degree of uncertainty.

For this purpose, the material data of PBT GF30 from [16] were utilized. PBT GF30 is a thermoplastic with short fibre reinforcement. The thermoplastic matrix material is reinforced with E-glass fibres, comprising 30 wt.% of the composite, with a nominal fibre diameter of 10 µm. It finds extensive application in the automotive industry due to its suitability for large-scale and cost-effective production through injection molding, as well as its excellent mechanical properties. The material exhibits anisotropic and nonlinear behavior. In [16], the fatigue behavior of this FRP was determined using the pearl-string method. A total of 34 measurements were conducted, divided evenly into the fibre orientations 0°, 45°, and 90°. The recorded 3D surface is presented in Figure 5, and the corresponding calibrated material data are listed in Table 1. The following formula describes the S-N surface as a function of the fibre orientation and the maximum number of load cycles:

$$\sigma_f(\theta) = \left[\frac{\cos(\theta)^2 \cdot (\cos(\theta)^2 - \sin(\theta)^2)}{\sigma_{\parallel,f}{}^2} + \frac{\sin(\theta)^4}{\sigma_{\perp,f}{}^2} + \frac{\cos(\theta)^2 \cdot \sin(\theta)^2}{\tau_{\parallel\perp,f}{}^2}\right]^{-\frac{1}{2}} \cdot N^b \tag{1}$$

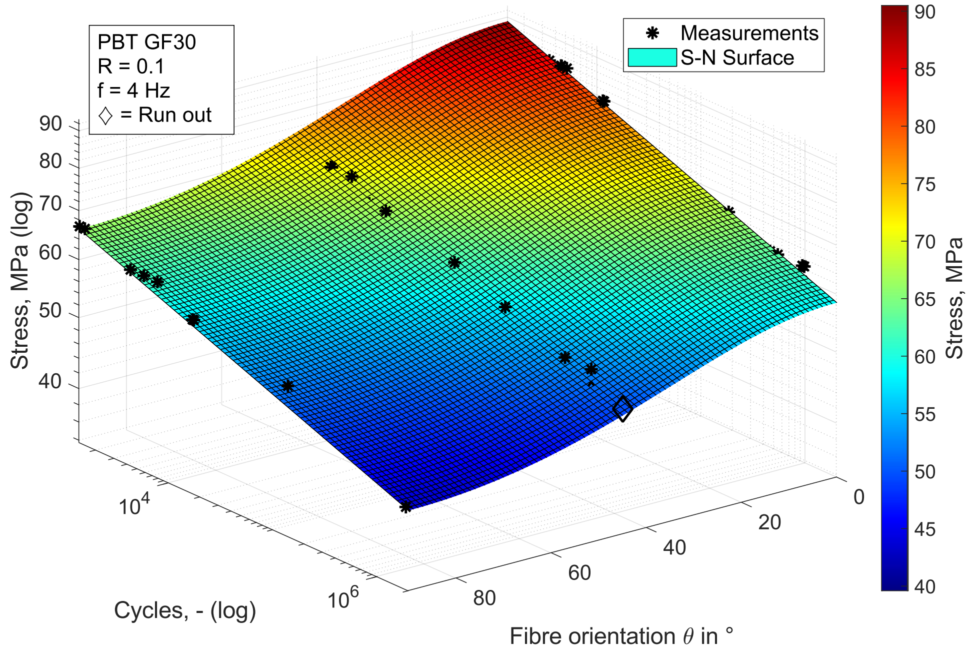

Figure 5: S-N surface interpolating arbitrary fibre orientations [16].

Table 1: Results for the calibrated S-N surface with upper/lower limit of 95% confidence interval [16].

Fatigue strength, parallel direction $\sigma_{\parallel}$	**132.9 MPa** (139.4 MPa, 126.3 MPa)
Fatigue strength, perpendicular direction $\sigma_{\perp}$	**102.3 MPa** (107.0 MPa, 97.6 MPa)
Fatigue shear strength $\tau_{\parallel\perp}$	**67.5 MPa** (71.1 MPa, 63.8 MPa)
Fatigue strength exponent b	**-0.057** (-0.053, -0.062)

The procedure for comparing the newly presented GP-based method with the pearl-string method is illustrated in Figure 6. In the first step, nine measurements are conducted to establish a baseline condition. Three measurements are taken for each of the fibre orientations 0°, 45° and 90°. They are distributed evenly across the high cycle fatigue range. Each measurement is obtained using Equation 1 along with the calibrated material parameters $\sigma_{\parallel}, \sigma_{\perp}, \tau_{\parallel\perp}$ and b from Table 1. An element of inaccuracy is introduced by utilizing a random value within the 95% confidence interval for each parameter, mirroring the experimental test conditions. These measurements serve as the initial baseline for both methods being compared, with each method then performing an additional 25 measurements.

For the pearl-string method, the combinations of load levels and fibre orientations to be tested are taken from [16]. To ensure a fair comparison between the two methods, the GP-based method is restricted to conducting its 25 measurements exclusively on the fibre orientations 0°, 45° and 90°.

The GP-based method employs a linear kernel to model the relationship between the applied load and the number of tolerable cycles, while the relationship between the fibre orientation and the number of tolerable cycles is modeled using a combination of a periodic and a squared exponential kernel.

Once both methods have selected their 25 design points, a curve fit is performed on the obtained results using Equation 1. The values derived from this process can be compared with those listed in Table 1. To obtain conclusive results, this procedure is repeated a total of 1000 times.

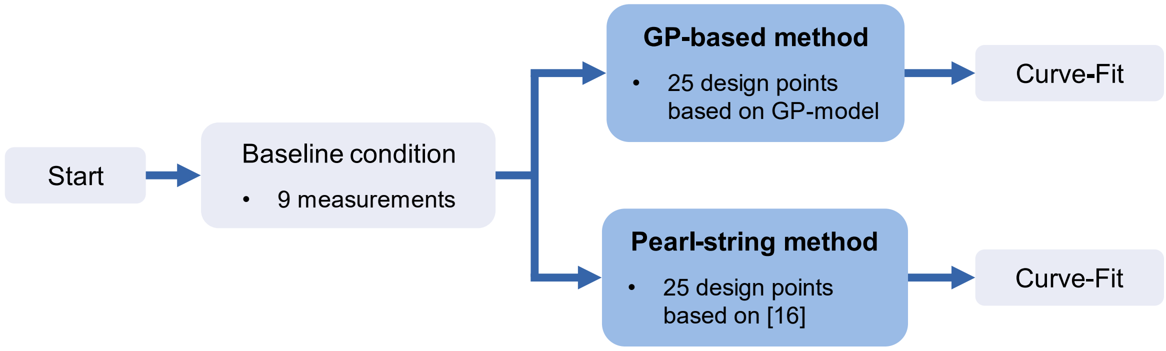

Figure 6: Principal procedure for comparing the GP-based method and the pearl-string method.

The results from the conducted test series are presented in Figure 7. The x-axis represents the deviation of the calculated material parameters from those listed in Table 1. Negative deviations were included on the positive side in terms of magnitude. The y-axis indicates the total number of runs corresponding to these deviations. Both methods exhibit a relatively symmetrical distribution in their results.

The pearl-string method shows a mean deviation of approximately 13.5%, while the GP-based method demonstrates a mean deviation of around 9.5%. The standard deviation for the pearl-string method is 6, which is higher than the GP-based method's deviation of 5.4. This suggests that the new method exhibits lower variability in the percentage deviation from the calibrated values. The presence of a distribution in both methods results from the assumption of random values from the 95% confidence interval for the strength values and the strength exponent in each measurement.

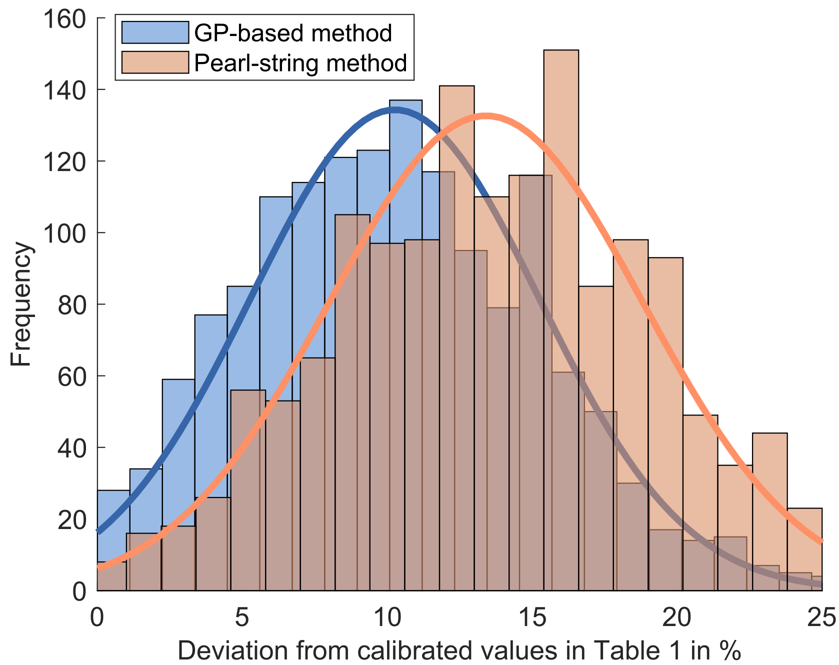

Figure 7: Histogram showing the results from comparing the GP-based and the pearl-string method.

The presence of a distribution in the results of the new method can also be attributed to the fact that selecting the point with the maximum information gain does not always guarantee a measurement with high information content. Under certain circumstances, the fluctuations assumed in equation 1 can lead to less informative measurements. On the other hand, the wider distribution in the results of the pearl-string method can be explained by the additional variation in selecting a design point with high information content, in addition to the inherent variation in the measurement itself. The new method performs better because it only has to deal with the inherent uncertainties in fatigue testing. In a direct comparison, the new method showed a smaller deviation than the pearl-string method in 75% of all cases.

6. Summary

The fatigue behavior of FRPs is a complex phenomenon influenced by a variety of factors. Existing experimental methods for fatigue characterization, such as the pearl-string or horizon method, simply vary the applied load when selecting the next design point. This approach results in an inefficient and incomplete characterization process, as other critical parameters are overlooked. A method based on Gaussian process regression was proposed to overcome this limitation. The necessary requirements for the use of the GP-based method in material characterization were also discussed. The newly presented approach was benchmarked against the pearl-string method in characterizing the fatigue behavior of the highly anisotropic FRP PBT GF30. The results demonstrate the superiority of the method, as it provides a more accurate understanding of the fatigue characteristics while only conducting the same number of experiments. Future efforts may focus on incorporating additional, economic factors into the acquisition function, such as considering the cost and time associated with fatigue tests. For example, a fatigue test with a cycle number of 10^6 is one-thousand times more expensive in terms of cost and time compared to one with 10^3 cycles. This would allow maximum information to be gained and experimental costs to be minimized.

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Author Contributions

Conceptualization, C.W.; methodology, C.W.; software, M.G.; validation, M.G. and C.W.; formal analysis, M.G.; investigation, M.G.; resources, M.G. and C.W.; data curation, C.W.; writing—original draft preparation, M.G.; writing—review and editing, C.W, T.H. and S.W.; visualization, M.G.; supervision, C.W. and S.W.; project administration, T.H. and S.W.; funding acquisition, S.W. All authors have read and agreed to the published version of the manuscript.

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