# 5 PRODUCT PORTFOLIO DECISIONS IN NON-DOMINATED FUNCTION SPACES

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Product portfolio optimization is the problem of providing the widest variety of functions while minimizing the product variety. Here we propose a novel approach based on functionality modeling for the product family, which can suggest portfolio decisions, building on significant dimensionality reductions in product variability. Function is modeled as a set of performance metrics reflecting the degree to which various user expectations are satisfied. If product variant A is better than (dominates) B on all counts, then A would be preferred; this implies that user preferences would lie among the "non-dominated" set of designs. Using a suitable multi-objective optimization algorithm, one may estimate this non-dominated set of designs, which restricts designs to a much lower-dimensional manifold in the function space. This non-dominated set which can then be clustered in an unsupervised manner, resulting in a candidate product groupings which the design team may inspect before arriving at a portfolio decision. We demonstrate the process on simple product platforms (faucets), in two scenarios: a) as an integral design, with continuous design variables, and b) as a 2-component design in which one component is available in several standard sizes. The effect of numerical stability in the process is investigated in empirically, and the conditions under which the results would scale to large dimensional spaces are also explored.

*Keywords:* Product Portfolio Optimization, Unsupervised Learning, Standardization.

## 1. FUNCTION IN PORTFOLIO PLANNING

Product portfolio selection is a key question facing the firm as it goes from design to manufacture — which of the products in a given line or product family should be the focus of manufacturing and marketing efforts? Arriving at a good portfolio, either in terms of a few standardized parts (integral design), or a few modular components,[22] leads to reduced inventory and efficient service, and crucially impinges on profitability. This problem has attracted growing attention in recent years,[12] and there has been considerable emphasis on minimizing the product variety, especially in terms of finding better commonality measures. In contrast, not much work has looked at function, partly due to the difficulty of modeling it. In this work, we assume that to a degree, some quantifiable models of performance may become available at late stages of conceptual design, and we show how these measures may be used to obtain a function space, which can lead to a simple algorithm for portfolio selection, with significant reductions in the dimensionality of the search space.

Here we take product family to be a set of products that serve a related set of market applications — they are similar in form and function, and share a common technology base, and may lead to similar processes for life-cycle design[10,11,16] (see Ref. 20 for a recent review). Given a product family, we take design space to refer to the common set of design variables within which the different product variants may be instantiated. This design space, especially for modular designs, is sometimes referred to as the product platform. Arriving at such a platform, especially for complex parts with many functional components, is a complex task, but for the purposes of this work, we assume that this is now available. There are two aspects of product portfolio selection. The first is to maximize the commonality between the parts, metrics for which have been the focus of a large body of work.[1,12,19,23] A second and relatively less modeled aspect is to consider the functional diversity among the objects. While work on part families have considered performance requirements to various degrees,[4,17,21] and other aspects such as manufacturing process design,[8] it has proved challenging to apply these ideas to portfolio standardization.

In this paper, we start with the assumption that by late stages of conceptual design, the functional requirements are sufficiently understood that they may be expressed in terms of quantifiable metrics. In order to validate these performance measures, clearly a considerable amount of prototyping, user validation and other measures would need to be taken. Also, over the lifetime of the product, the degree to which these functions reflect actual performance keeps improving. Also, new functions may be added, resulting in different product groups forking off based on these differences.[3] However, for the purposes of this work, we simply assume that some reasonable estimates are available. Now we can then identify certain user preferences as inferior if they are poorer in all the functional measures. Given a set of k performance criteria f1, f2, …, fk, We say that product variant A dominates variant B if the fi(A) is superior to fi(B) for all k performance metrics. Then, we may assume that user preferences would lie among the designs which constitute the non-dominated set, which amounts to a vast pruning of the design space. Finally, we show how given this non-domination set, one may define product similarity metrics, which can be used for clustering the parts in an unsupervised manner. These similarity measures may be defined on the design space variables, functional aspects, and also other measures of commonality proposed in the literature.[1,14] This results in product groupings in the design space, each of which reflects product similarity (in terms of design space as well as function). The design team may then consider these groupings as candidate product classes, each of which may be represented by a single exemplar or product variant, the set of which would constitute the product portfolio.

One of the challenges of portfolio optimization is that the search space is very large, a consequence of the high dimensionality (the number of parameters N) in the design space. For example, for a product such as a mobile phone, $N$ may be about 50. Of these, about 20 may be discrete, say with 4 levels each, and 30 may be continuous (at least 10 values each, say). Then the search space is O(1042), which is simply too unmanageable. The first advantage of this approach is that it dramatically reduces the dimensionality of the search space; this is because the designs are limited to those in the non-dominated front, which is a surface in the space of functional measures. If there are $k$ function measures (usually $k \ll N$) then the feasible set of designs lie on a $k-1$-manifold, which is a dramatic reduction in the search space dimensionality. For discrete variables, the designs may lie on some bands in this manifold, further reducing variability.

Another important advantage is that it is computable entirely in simulation, and does not require the market data commonly assumed by product portfolio design tasks.[2,9] Indeed, obtaining such market data would require some initial manufacturing, which would already tie in some of the design decisions, and result in potentially significant opportunity loss.[18] Part of the reason that firms are unable to forego such risks is the forbidding nature of the search space outlined above.

The process outlined here is based on a view of design which considers the design process in terms of the design space (or product platform) D, the actual design variants or structures S, their set of behaviours B, on which are defined a set of performance measures P. This DSBP model is defined in Section 2. Once the Performance measures are available, we may use any Multi-Objective optimization technique (we use NSGAII,[5] a well known evolutionary algorithm), to estimate the non-domination front (Section 3.1). Finally, the instances found in the non-domination front can be clustered using any unsupervised classification algorithm (k-windows, dbscan, neural gas); we use a variant of neural gas called Growing Neural Gas.

## 2. MODELING FUNCTION

Our design process (Figure 1) involves a cycle through Design — Structure — Behaviour and Performance (D-S-B-P). The Design space D is the space of all design variables (each design is a unique design vector), and initial constraints imposed as part of the design specifications. The Structural space S is the space of the parameters needed to completely define the geometric and material structure; there is an one-to-one mapping from the design vector v, to the structural vector. Given the structure, a large set of behaviours B are possible, of which only a few, the performative behaviours BP, are of interest for functional purposes. Finally, a set of metrics maps the behaviours in BP, to an evaluation space P, where the performance metrics ($pj$, $j = 1 \ldots k$) are orthogonal to each other. The results of evaluation are then used to search in design space and come up with an improved design in D.[4]

## 3. EXAMPLE: WATER FAUCET

As an example task, we now take up the detailed design of a basin faucet modeled using simple geometric elements for the inlet, outlet and knob. Each of these design elements has a set of driving geometric parameters; in terms of which all other shape parameters as well as joining constraints can be defined. Also the design space for overall product family may have up to 20 parameters, in the analysis below, we restrict the driving parameters to three for the ease of demonstration in this paper.

In this work, we consider two different types of product family. One is integral — i.e. all the shape parameters are continuous and constitute a single design space. in the second model, the product is modular, and two of the components (the knob/inlet) are available in three standardized sizes, 8, 9, and 11 cms. We show how the same process can be used in both these situations. Even when the design space is discrete (modular), the clusters need not follow this discrete variation, it may cut through the design space in other ways as well.

### 3.1. Estimating the Non-Dominated Front

Design is a search process. During this process designers tries to find optimum solutions through searching the design space. The member of the part family is characterized by a set of design variables. In which we focus on a 5-tuple design vector *w, L,* •1 and •2 which we call as driving variables as the other design dimensions internal to the faucet Figure 2 are defined in terms of these driving variables. For example, the height *(h)* of the base is 0.5 L. Given a set of values for a design vector, one can determine its shape. The optimal solutions can be obtained by modeling the above problem as a multi-objective optimization problem for searching the design space and we have used NSGA-II algorithm.[5]

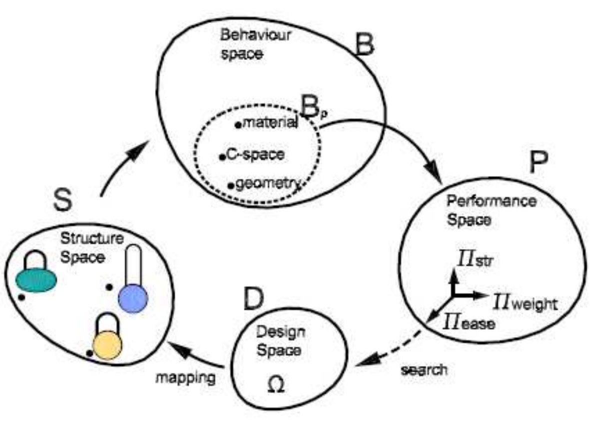



**Figure 1.** Design Process: A mapping from design space to structure space and searching the design space based on performance evaluation of the intended behaviours.

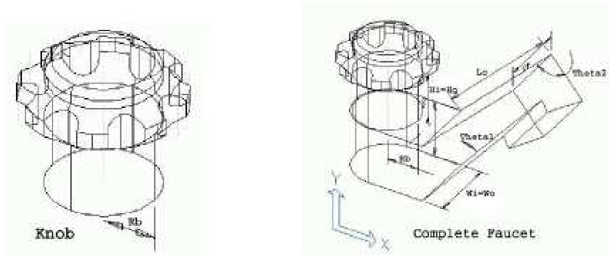



**Figure 2.** Complete Faucet with Knob. The driving parameter set {Wo, Ho, Lo, $\theta 1$, $\theta 2$} And the Knob radius {Rb} is 0.5Wo.

**Multi-Objective Optimization**

$$
\begin{array}{llll}
\text{Maximize} & \pi_1(\underline{v}) = C_d AV, A = wh & V = \sqrt{2gH_{net}} & \\
\text{Minimize} & \pi_2(\underline{v}) = \frac{\rho v_{vol}}{L^2}, & & \\
\text{Subject to} & g(\underline{v}) \equiv 55.0 < \theta_1 < 60.0, & 5.0 < w, h < 8.0 & \\
 & 20.0 < L < 40.0, & 70.0 < \theta < 150.0 & \\
 & \theta_2 = \frac{55.0w - 20.0}{3.0} & \theta_1 = 0.5L + 40.0 & \\
 & h = w + 0.5 & &
\end{array}
\quad (1)
$$

Where $H_{net} = ((H - L\cos(\theta_1) - (0.85h)\cos((\theta_1 - \theta_2)) - H_f))$

$$H = 1000 \quad \text{and} \quad H_f = 0.8H, \quad V_{vol} = V_{tap}$$

$$V_{tap} = V_{body} + V_{spout} + V_{mouth} \qquad V_{body} = \frac{\pi w^2}{2} h + wh^2 \qquad V_{spout} = 0.5w(L^2 \sin(\theta_1))$$

$$\sin(\theta_0) + L^2 \cos(\theta_0)\sin(\theta_0) + wL\cos(\theta_0) \qquad V_{mouth} = 0.8h^2 w sin\left(\frac{\theta_2}{2}\right)$$

## 3.2. Discovering Product Subcategories

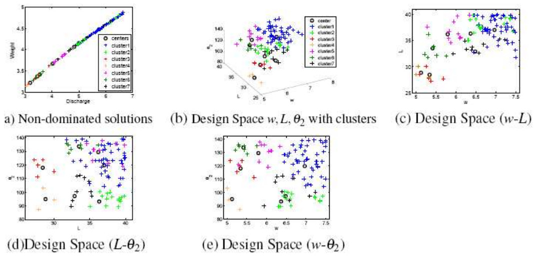
Here we demonstrate the process of obtaining clusters in the design space based on the user preferences in the form of performance behaviours. With suitable metrics on these performance behaviours and the search in the design space based on these metrics we can come up with a set of Pareto optimal solutions in the space high dimensional performance measures. In this paper, we consider performance metrics (user preferences) to be optimized may be defined over some continuous and discrete variables. For the faucet, the knob can have discrete radius R values to fit into the inlet of the tap. First, we present the clusters for continuous variables and then we show the standard parts for the discrete variables.

In order to find out the clusters based on product similarities we use un-supervised learning algorithms in which the products (design vectors in design space W) can be clustered in an unsupervised manner based on the notion of distance between products. We use the Growing Neural Gas (GNG) algorithm[7] for unsupervised clustering which has been shown well suited for finding clusters.

### 3.2.1. Product grouping for integral Part families

For the integral faucet part families, the design parameters w, L, and $\theta 2$ are continuous and spanning the design space shown in Figure 3(b). Our interest is to find out the clusters in the design space of the non-dominated solutions. The Figure 3(a) shows the non-dominated front for bi-objective functions involving three independent design variables constitutes the design space. With the help of unsupervised clustering we show the clusters in Figure 3 and also the data distribution in the projected design spaces w-L, w-$\theta 2$ and L-$\theta 2$.

In the case of faucets we considered here, each faucet will have knob,inlet and outlet. We can create different faucets with different types of faucet components. Let us say, for the two components knob and inlet, if the radius R is available in different standardized sizes 5, 8 and 11 cms, different



a) Non-dominated solutions (b) Design Space $w, L, \theta_2$ with clusters (c) Design Space ($w$-$L$)

(d)Design Space ($L$-$\theta_2$) (e) Design Space ($w$-$\theta_2$)

**Figure 3.** *Product groupings for faucet based on non-domination front:* The non-dominated solutions (a) are mapped into the design space (b) where clusters are obtained using unsupeverised clustering. Here three clusters are shown in different colours for data input of 500. Note that some clusters obtained from the design space overlap along the Pareto front. The large circles are the nominal cluster centers. *2D Sectional Distributions* The projections of clusters are shown in design subspaces (c) $(w, L)$, (d) $(L, \theta_2)$, and (e) $(w, \theta_2)$.

**Table 1.** Cluster centers for the data input of 500 and the corresponding design parameters' values.

| Clusters | $w$ | $L$ | $\theta_2$ |
|---|---|---|---|
| C1 | 6.96 | 37.02 | 119.71 |
| C2 | 6.36 | 36.33 | 93.2 |
| C3 | 5.33 | 28.46 | 117.9 |
| C4 | 5.12 | 28.85 | 94.98 |
| C5 | 5.79 | 36.20 | 129.43 |
| C6 | 5.41 | 33.51 | 133.63 |
| C7 | 6.48 | 32.89 | 98.18 |

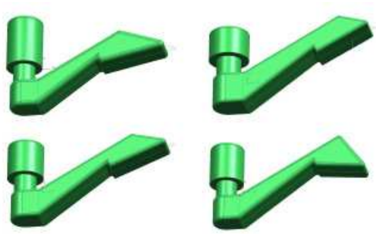
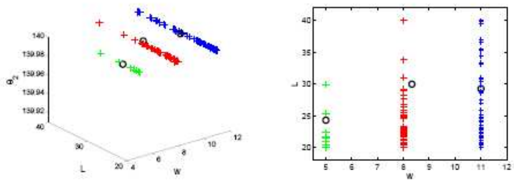


**Figure 4.** *Faucet Family*: Four of the product groups from Table 1, corresponding to the centers of clusters C1, C2, C4, C5.

combinations of faucets can be created for higher variety. When we use unsupervised clustering in the case of discrete variables, the clustering follow based on discrete variation. Figure 5 shows the design space with different clusters.

## 4. SCALABILITY

The approach presented here makes two claims that (a) user preferences can be estimated a priori given good estimates of product function, and that (b) the non-dominated set of designs results in significant pruning in the product search space, especially where the dimensionality of the product design variables N is much higher than the number of distinct functional considerations. Other aspects of the approach, such as the similarity measure used in unsupervised clustering, remain matters of detailing, on the whole, if these two assumptions hold, it is likely that the approach presented may constitute a valuable portfolio optimization tool.



(a) (b)

**Figure 5.** *Design space with different standard sizes of w 5,8 and 11 cms.* (a) Three clusters are shown in three colors in the design space $w, L$ and $\theta_2$ and the same clusters are shown in the design sub-space $w, L$.

The approach presented has been validated on a very simple, indeed, toy part family, consisting of faucets that vary in only three independent dimensions. The question immediately arises as to whether this technique would be viable for much larger design spaces, for example those involving real products such as cameras or mobile phones or aero planes, in which tens or even hundreds of components, each with several dozen design variables, are combined and the final product is evaluated under dozens of functional considerations.

We now explore these two assumptions for large design spaces. The first assumption, that quantifiable models of functional performance may be available at the late stages of design, is open to challenge. First of all, many of these functions may not be available as analytic expressions in the design variables, but the relationship may be implicit, or available only as a result of simulation, or may even involve (e.g. in matters such as aesthetics), a degree of subjective judgment. However, designers usually have some idea of the set of functions they are working with, and arriving at such a set of functions may not be impossible.

In practice, performance measures improve considerably with increased experience. More significantly, new and unanticipated functions arise. Also, the quantity that the functional metrics estimate, consumer preference, itself changes with increased exposure to the product. Thus, on several fronts, it is clear that any initial estimate of function is at best a poor approximation. However, having said this, it remains the best one can do given the available information, and surely it still remains a better alternative than to actually manufacture some inventory of products based on some uninformed guess, and use this experience to re-formulate the portfolio.

The second assumption, that restricting our attention to the non-dominated set of designs constitutes a significant pruning of the design space, requires some exploration. The Design Space W is a finite set of design variables vi, $i = 1 \ldots N$, and the function space P has a set of performance metrics pj, $j = 1 \ldots k$. Let us assume that the functions are non-complimentary, so that a non-domination front exists as a k−1-manifold F in the function space P. Now, restricting our attention to designs in the non-dominated set implies that $p$( ~(v)) 2 F. Clearly, this constitutes an additional restriction on ~($v$), and thus bounds it more tightly than~($v$) 2 W, but we would like to know whether this reduction is significant.

We observe that the dimensionality of the function space in practice is usually much less than that of the design space, i.e. $k \ll n$. For example, a digital camera (in terms of all its components, and assembly processes) may have several hundred parameters, but only about ten functional measures. Thus, on the face of it, the design seems to be constrained to a much lower dimensional manifold. However, this explanation is naive, for the functional measures, *fj*(~($v$)), as pointed out earlier, may be non-analytic, complex and nonlinear.

To consider a particular case in detail, let us assume that the variables are continuous, and that an explicit formulation of function is available. Then we may evaluate the variability of ~($v$) in terms of the variability in~(f) by considering the Jacobian J. If J is close to being singular (i.e. its rank is close to zero), then designs that are very similar in function (low Ñ~($f$)) may arise from design variables that are conspicuously different. In this situation, designs within a lower-dimensional manifold in the function space may cover a large swath of the design space. If the Jacobian is not ill-posed, the assumption is likely to hold.

For most design tasks, if a small change in the design parameters results in very large changes in function, such designs would consider unstable. However, whether the inverse holds — where very widely differing designs result in similar functions may not be well known. However, considering that there is more than one functional measure, it is unlikely that all of these would coincide for widely varying design parameters. Thus, on the face of it, it is likely that most such jacobians would be well-posed, and hence the non-dominated set would occupy, especially for high N, a relatively sparse region in the design space.

The computational aspect of scalability cannot be assessed at this point, since neither the nature of the performance metrics nor the process of non-linear optimization can be precisely characterized in terms of any measure of problem size. While it may happen that some of the functions involved may involve considerable complexity, it is unlikely that other approaches would improve on this one because of the dramatic pruning in the design space achieved by restricting our focus on the non-dominated set.

## 5. CONCLUSION

In this work, we have presented an approach to the determination of product portfolio's based on function. We have assumed that some quantifiable estimate of the user's preference is available, and shown that the resulting non-dominated set usually occupies a relatively sparse region in the design space. We have outlined a computational procedure for obtaining clusters in product space by focusing only on the non-dominated set of designs. These clusters may serve as candidate groups, which can be used to obtain a product portfolio.

As a test case, we have considered both integral designs with continuous variables, and also modular designs with some discrete variables, and shown that the same approach works in both situations. We have also presented an empirical demonstration, by choosing differing data densities in the non-domination front, that the results may be relatively stable against numerical variation. However, much work remains in this area, of discrete choices, and component-based modularity. Although the demonstration was on a very simple problem, we have argued that the process is likely to be scalable. However, what the limits are, and the extent to which the process can be applied on real design situations will only be known with time.

## REFERENCES


[1] Khajavirad, A. and Michalek, J. 2007 An extension of the commonality index for product family optimization. in DETC2007 (September), pp. 4–7.

[2] Bruno Agard and Andrew Kusiak. 2004 Standardization of Components, Products and Processes with Data Mining. in International Conference on Production Research Americas (August).

[3] Madan Mohan Dabbeeru and Amitabha Mukerjee. Discovering Implicit Constraints in Design. In Third International Conference on Design Computing and Cognition (Atlanta Georgia USA, June 2008), Springer.

[4] Madan Mohan Dabbeeru, and Amitabha Mukerjee. 2008 Functional Part Families and Design Change for Mechanical Assemblies. In Proceedings of DETC'08 (August 3–6 2008), ASME Design Engineering Technical Conferences.

[5] Kalyanmoy Deb. (2001). Multi-Objective Optimization using Evolutionary Algorithms, 1 ed. Chichester, John Wiley and Sons, Ltd.

[6] Kalyanmoy Deb and Aravind Srinivasan (2007). INNOVIZATION: Innovative Design Principles Through Optimization. Tech. Rep. Kangal:2005007, Indian Institute of Technology Kanpur, Kangal, IIT Kanpur.

[7] Bernd Fritzke. (1995). A Growing Neural Gas Network Learns Topologies. *Advances in Neural Information Processing Systems*, 625–632.

[8] Zhengdong Huang and Derek Yip-Hoi(2003). Parametric Modeling of Part Family Machining Process Plans From Independently Generated Product Data Sets. *Journal of Computing and Information Science in Engineering* (July 2003).

[9] Jiao, J., Zhang, Y. and Wang, Y. (2007). A heuristic genetic algorithm for product portfolio planning. *Computers and Operations Research*, 34(6):1777–1799.

[10] Jianxin Jiao, Qinhai, M. A. and Mitchell M. Tseng (1998). Design For Mass Customization By Devloping Product Family Architecture. In Proceedings of DETC'98 (September 13–16 1998), ASME Design Engineering Technical Conferences.

[11] Jianxin Jiao, Qinhai, M. A. and Mitchell M. Tseng (1998). Towards High Value added Products and Services: Mass Customization and Beyond. *Technovation*, 23(10):809–821.

[12] Jianxin Jiao, Timothy W. Simpson and Zahed Siddique (2006). Product Family Design and Platform-Based Product Development: A State-of-the-Art Review. *Journal of Intelligent Manufacturing*.

[13] Kannan, B. K. and Kramer, S. N. (1994). An augmented Lagrange multiplier based method for mixed integer discrete continuous optimization and its applications to Mechanical Design. *Journal of mechanical design*, 116(2):405–411.

[14] Kota, S., Sethuraman, K. and Miller, R. (2000). A metric for evaluating design commonality in product families. *Journal of Mechanical Design*, 122(4):403–410.

[15] Messac and Mattson, C. A. (2004). Normal Constraint Method with Guarantee of Even Representation of Complete Pareto Frontier. *AIAA JOURNAL*, 42(10):2101–2111.

[16] Meyer, M. H. and Utterback, J. M. (1993). The product family and The Dynamics of Core Capability. *Sloan Management Review*, 34(3):29–47.

[17] Raviraj U. Nayak, Wei Chen and Timothy W. Simpson. (2002). A Variation-Based Method for Product Family Design. *Engineering Imization*, 34:65–81.

[18] Newnes, L. B. (2008). Predicting the whole-life cost of a product at the conceptualdesign stage. *Journal of Engineering Design*, 19(2):99–112.

[19] Kota, S., Sethuraman, K. and Raymond Miller (2000). A Metric for Evaluating Design Commonality in Product Families. *Journal of Mechanical Design, ASME*, 122:143–150.

[20] Timothy W. Simpson (2004). Product platform design and customization: Status and promise. *Artificial Intelligence for Engineering Design, Analysis and Manufacturing*, 18:3–20.

[21] Timothy W. Simpson, Jonathan R. Maier and Farrokh Mistree (2001). Product platform design: Method and application. *Research in Engineering Design*, 13(1):2–22.

[22] KarL Ulrich(1995). The role of product architecture in the manufacturing firm. *Research Policy*, 24:419–440 (May 1995).

[23] John G. Wacker and Mark Treleven(1986). Component Part Standardization: An Analysis of Commonality Sources and Indices. *Journal of Operations Management*, 6(2):219–244 (February 1986).