



## OPTIMUM DESIGN OF A FORMULA ONE REAR WING USING A GENETIC ALGORITHM

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### 1. Introduction

Due to their high strength-to-weight ratio and the possibility of tailoring their stiffness by selecting fibre orientations, fibre-reinforced composites are often more profitable than conventional materials. As a result their use in mechanical, aerospace, automobile, shipbuilding, and other branches of engineering, is on the increase [1] and because of the strong development of affordable fabrication technologies and commercialisation, the demand for these materials is expected to rise considerably in the near future.

In this context, new effective methods in the optimum design of composite structures are increasingly required. Many researchers have developed techniques based on strength, stiffness and manufacturability optimization for determining the lay up of composite laminates [2].

For the practical problems in which the ply thickness is fixed, the design of composite structures becomes a discrete optimisation problem (with ply orientation angles as design variables), which is suitable for genetic algorithms. Genetic algorithms (GAs) have recently attracted attention for solving optimization problems [3], and there have been several papers on optimization of composite structures by the use of this method [4-6]. GAs are based on the mechanics of natural selection and genetics and seek the optimal solution through random probability methods without auxiliary conditions such as continuity of the variables, and intelligently chosen starting points. These conditions are typical of traditional searching techniques such as gradient-based techniques.

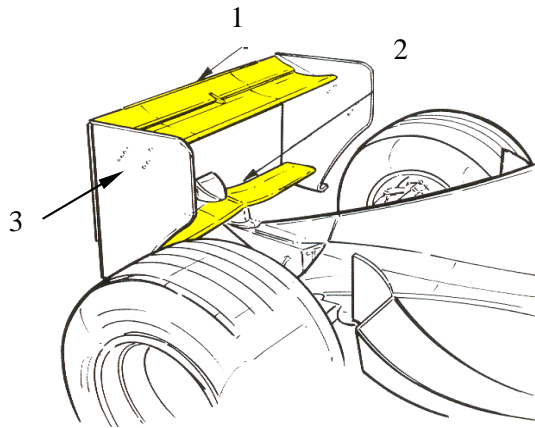
In the present paper a genetic algorithm was applied to find the optimal stacking sequence of laminates constituting a Formula One lower-rear wing. The genetic algorithm, developed on a personal computer by using the analysis program *Mathematica* [7], applies the general-purpose finite element code NASTRAN to carry out the stress analysis.

### 2. Basic of rear wing

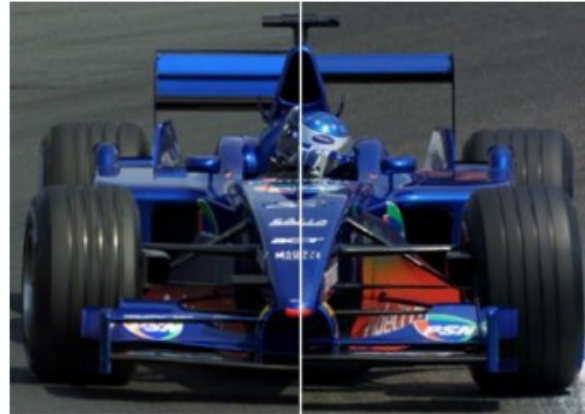
There are many factors influencing the performance of a Formula One car, and current changes concentrate on the mechanical grip of the car. However, it is an acknowledged fact that aerodynamics plays a big part in the problems of overtaking, and since the introduction of aerodynamic principles into Formula One, they have become of ever increasing importance to the car's performance.

The aerodynamic requirements of the F1 car are high downforce at minimum drag. The wings on an F1 car produce this downforce. They use the same principle as those found on a common aircraft, but while the aircraft wings are designed to produce lift, wings on an F1 car are placed 'upside down', to produce downforce and push the car onto the track.

About a third of the car's total downforce can come from the rear wing assembly. This device (Figure 1) is the one that is varied the most from track to track. As the rear wings of the car create the most



**Figure 1. A rear wing assembly**



**Figure 2. Difference in the rear wing during different GP**

drag, the teams tailor the rear aerodynamic load to suit a particular track configuration. Monza in Italy is a very fast track with long straights and few corners, with full throttle being achieved for around 70% of the lap. As more wing angle creates more downforce, more drag is produced, reducing the top speed of the car. At Monza therefore, top speed is vital, so teams run very little rear wing angle to reduce drag (Figure 2- right). At Monaco, where the car is constantly turning in and out of corners, downforce is vital, so maximum downforce is needed from the wings (Figure 2- left).

The rear wing is made up of two sets of aerofoils connected to each other by the wing endplates. The top aerofoil (number 1 in figure 1) provides most of the downforce and is the one that is varied the most from track to track. It is now made up of a maximum of three elements due to the new 2001 regulations. The lower aerofoil (2) is smaller and is made up of just one element. As well as creating downforce itself, the low-pressure region immediately below the wing helps suck air through the diffuser, gaining more downforce under the car. The endplates (3) connect the two wings and prevent air from spilling over the sides of the wings, maximising the high-pressure zone above the wing, creating maximum downforce.

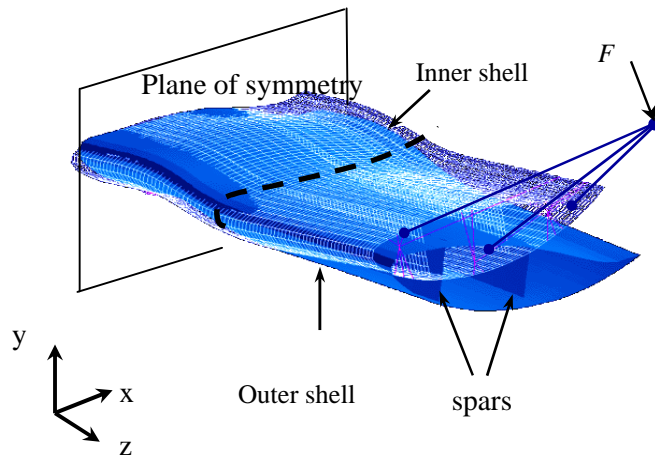
### 3. Lower wing structural design

The lower wing is certainly the rear wing element subjected to the worst loading conditions during the competition. In addition to the aerodynamic pressure, the lower wing supports the loads operating on the upper wings to which it is strictly connected. These actions result in twisting and bending effects, whose intensities must be limited to avoid the loss of the aerodynamic performance required by the team.

The structural design of the lower wing in a shape which satisfies aerodynamic requirements, then becomes a stiffness optimisation problem. The aim of this optimization is to find the best laminate stacking sequences, in a trade-off between stiffness requirements and weight reduction, without laminate failure due to excessive stress. For a fixed value of ply thickness and a small set of fibre orientations, the problem of laminate stacking sequence design is

**Table 1. Material properties**

	T800	M46J
longitudinal modulus of elasticity	70 GPa	245 GPa
lateral modulus of elasticity	70 GPa	5 GPa
transverse shear modulus	5 GPa	5 GPa
in-plane shear modulus	7 GPa	5 GPa
major Poisson's ratio	0.45	0.35
long. allowable stress in tension	1300 MPa	2180 MPa
long. allowable stress in comp.	1300 MPa	50 MPa
lateral allowable stress in tension	840 MPa	1200 MPa
lateral allowable stress in comp.	840 MPa	170 MPa
allowable stress in-plane shear	138 MPa	64 MPa
allowable interlaminar shear stress	85 MPa	64 MPa
mass density	1.59 g/cm <sup>3</sup>	1.61 g/cm <sup>3</sup>
ply thickness	0.18 mm	0.12 mm



**Figure 3. Schematic representation of the lower wing**

analysis using the general-purpose finite element code NASTRAN, with MSC.Patran 2000 as the pre- and post-processor. The finite element model of the rear wing was constructed using 6000 elements (CQUAD4 and CTRIA3). Because of geometrical and loading symmetries only one half of the structure was considered, introducing suitable continuity constraints on the cut plane.

The aerodynamic loads at the maximum speed were simulated by an equivalent load of about 3550 N oriented at an angle of 30 degrees with respect to the y-axis. The equivalent load was evaluated by measuring in the wind tunnel. The connection hinges were simulated using three rigid bars.

In figure 3 there is a schematic representation of the rear wing before and after the load application.

#### 4. Algorithm of Optimisation

GAs are probabilistic search techniques based on the mechanics of natural selection and natural genetics. They solve optimisation problems, imitating nature in the way it has been working for millions of years in the evolution of life. The stronger individuals in a population are more likely to survive than the weaker; they can generate offspring and transmit their heredity to new generations. Nature tends to preserve the chromosomes that cause beneficial adaptations to a given environment, and introduces variation in a species when reproduction occurs.

Genetic algorithms have received considerable attention in recent years in an effort to better understand their search characteristics, capabilities and computational efficiency. They appear well suited to solve problems involving large and discrete search spaces, where the gradient-based techniques are not very effective; but their application usually involves very large computational costs. A simple genetic algorithm involves a set of individuals (population), and a set of genetic operators. Each individual in the population represents a design, i.e. a stacking sequence in the code of a string. The genetic operators allow the genetic manipulation process (reproduction) be carried out.

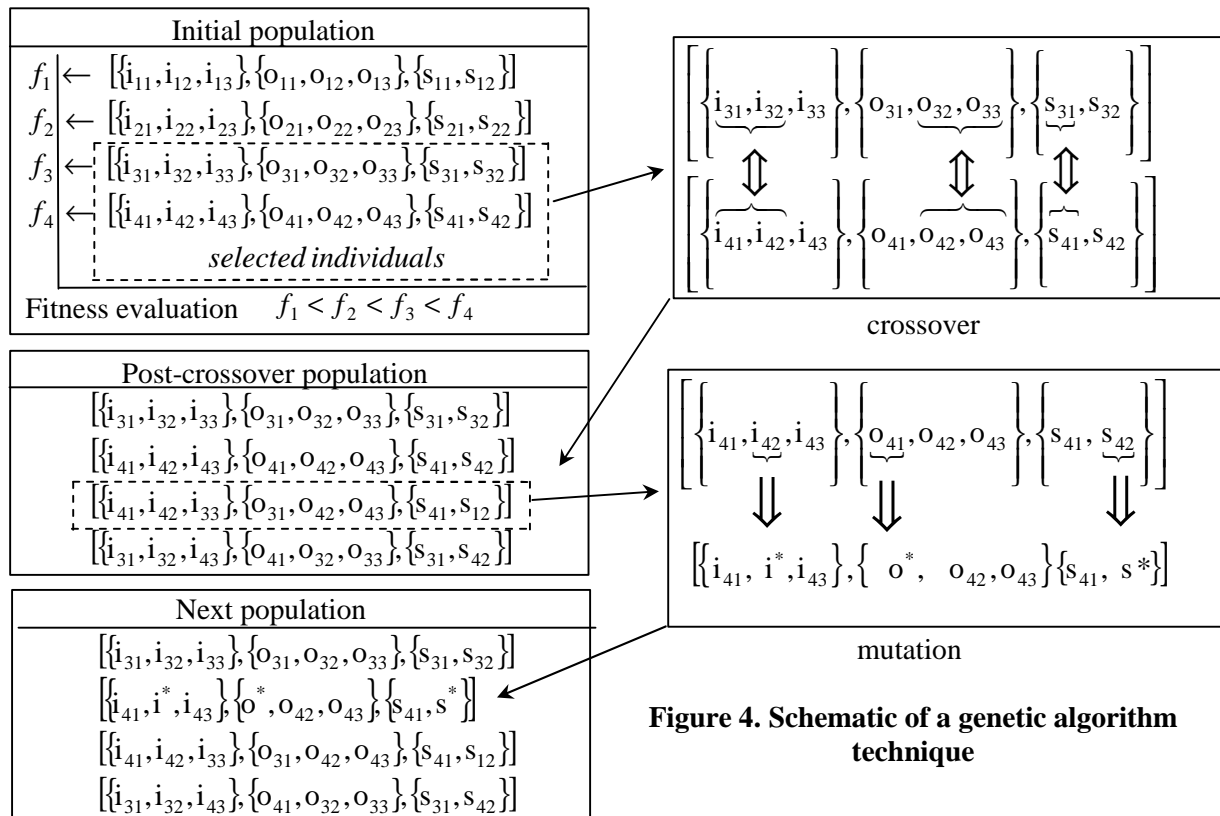
A GA begins with the random generation of a population (initial population) of potential solutions. By means of a randomized process of selection based on the objective function values, individuals for the reproduction are chosen. The fitness of each design in the population is evaluated by computing the value of the objective function, and the selection process permits those individuals of superior fitness to reproduce more than others. Selected designs are, successively, processed by means of the genetic operators to create a new population, which combines the desirable characteristics of the old population. Then the new population replaces the old one and the process restarts.

The reproduction is generally based on two operators; crossover and mutation. The crossover operation exchanges a partial set of attributes between selected design pairs, based on a crossover probability,  $p_c$ , to generate new offspring.

discrete in nature. Therefore, the design of the stacking sequence is a combinatorial optimisation problem that is suitable for genetic algorithms.

To investigate this possibility, in the present paper, a composite lower wing 100 cm long with a wingspread chord of 15 cm was optimised. Deflection and twist angle to the wingtip were assumed as stiffness parameters.

M46J Graphite/Epoxy unidirectional plies and T800 Graphite/Epoxy woven fabric plies were considered to make each individual part of the wing. Material properties and ply thickness are given in Tables 1. The determination of stiffness parameters for each possible solution was carried out by a numerical



**Figure 4. Schematic of a genetic algorithm technique**

The mutation operation permits exploration of other areas in the design search space by probabilistically altering the genes on a chromosome in the current population. The probability of mutation,  $p_m$ , is used to determine the number of mutants that are to be introduced into the current population. New generations of designs are created through the genetic manipulation, and this iterative process is repeated for a fixed number of generations or for a fixed number of analyses without improvement in the best design.

A finite length string must represent each individual (chromosome) of the population. Usually, binary strings have been used for this purpose. Dealing with composite structures, a laminate can be directly coded using the standard stacking sequence notation, i.e. a string of ply orientation angles such as  $[0^\circ, 45^\circ, -45^\circ, 90^\circ]$ . This choice allows the simplification of the algorithm implementation and, moreover, make it possible to investigate the optimisation problem, with great straightforwardness, choosing the design variables among different sets of permissible ply orientations.

Figure 4 presents the application of the simple genetic algorithm to an illustrative case, with a population of four designs. Each design is constructed by assembling all the laminate stacking sequences (in the case of the lower wing examined,  $i_{ij}$ ,  $o_{ij}$ , and  $s_{ij}$  can represent, for example, the plies of the inner-shell, the outer-shell and the spars respectively).

Consider the case where the four designs have their objective function ( $f$ ) ordered as  $f_4 > f_3 > f_2 > f_1$ , by virtue of which, designs 4 and 3 are the fittest design in the population.

Selection is accomplished using the steady state strategy which to obtain the new population substitutes only a certain number of individuals of the old population with their descendants, for example the worst half of the population. Note how the selected population in figure 4 contains, designs 4 and 3, which have the larger  $f$  values. Furthermore, note that design 1 and design 2 which have the least values of the objective function are not propagated to the succeeding generations.

The selected population undergoes the three-point crossover operation between probabilistically selected design pairs, as exemplified by the design pairs 3-4 in figure 4. In this step, selected portions of the stacking sequences representing the selected designs are swapped as shown in the crossover inset in the figure. Afterwards, a mutation operation is performed on the post-crossover population; where the designs selected based on the mutation probability undergo a complete regeneration. This operation is shown schematically in the mutation inset in figure 4. The resulting population constitutes

the next generation. One generation after another is created until the convergence criteria discussed above are met.

As is evident from the foregoing discussion, the parameters involved in the genetic algorithm are the population size, the number of generations, the probability of crossover, and the probability of mutation. The values of these parameters are problem-specific, and are selected on the basis of systematic trials aimed at the efficient performance from the algorithm.

## 5. Results

A genetic algorithm used for the rear wing optimisation was developed on a personal computer by using the analysis program *Mathematica*. The implementation of the algorithm is shown in figure 5, and GA parameters are given in Tables 2.

The process starts with the generation of a random initial population of laminate stacking sequences. Each design is randomly formed by choosing the ply orientation angles inside a set of values given by the user. Within this set the user can introduce the acronym *np* (no ply) for including the possibility of ply suppression.

For each design of the population, in the NASTRAN pre-processor stage, the MSC/NASTRAN input file created by the MSC/PATRAN translator is adjusted, by modifying PCOMP bulk data entries, to take the proper layout of composite laminates into account. Then the FEM analysis is carried out.

In the NASTRAN post-processing stage, the stiffness parameters are evaluated and saved. After that, all the FEM output files are removed to release the computer memory and, the cycle restarts. The fitness processor begins to operate at the end of the population processing, evaluating the objective function for each design. The most suitable solutions are selected and then processed by means of the genetic operators to create the new population. The process is repeated until convergence.

The goal of the optimisation is to find the thinnest (minimum weight) stacking sequence to which corresponds the smallest vertical deflection to the tip-wing without any laminate failure due to excessive stress. The stacking sequence is also constrained to have no more than three contiguous plies with the same orientation to avoid problems with matrix cracking.

The objective function identified for this optimization problem is:  $f = \frac{1}{a y + b W + c}$  where  $y$

represents deflection to the wing-tip,  $W$  is the weight of the entire composite structure,  $a$  and  $b$  are two weighting factors with values chosen in agreement with search requirements. The symbol  $c$  is a penalty function, whose values range between 0 and 1, that is applied for failure or for more than three contiguous plies presence,  $c = 0$  when no failures and no contiguous plies are present. Ply failures are detected by the monitoring failure index based on the Hill criterion.

The variation in fitness value with the number of generations during the optimization process is plotted during the run on graphs similar to that shown in figure 6. The maximum fitness value is the

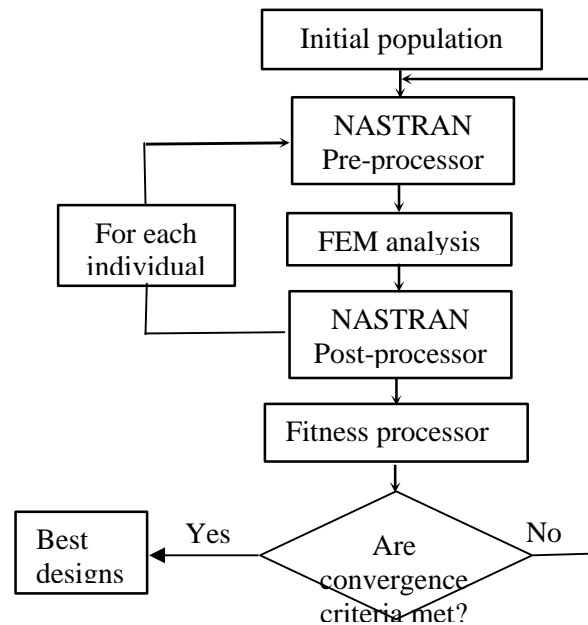


Figure 5. GA implementation

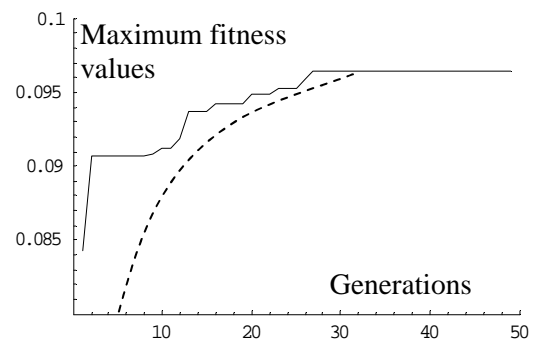
Table 2. GA-parameters

Population size	40
Length of chromosome	48
Selection strategy	Steady state
Crossover strategy	$p_c = 0.7$
Mutation strategy	$p_m = 0.05$

best parameter among the population in each population (continuous line). On the same graph the average fitness value (dotted line) is also reported. The average fitness value is the average of all the fitness values of the population. In earlier generations, the value of average fitness will be less since the population consists of the worst individuals. Over the generations, the population becomes composed of fitter individuals, with only slight deviations from the fitness of the best individual so far found. Hence, the average fitness comes very close to the minimum fitness value.

To verify its effectiveness, the algorithm was carried out using 21, 16 and 8 unidirectional plies for inner shell, outer shell, and spars respectively. Skins in woven fabric coated the multidirectional laminates externally. Although woven fabric has lower structural capabilities than unidirectional composite, (as the lower value of the Young modulus reported in Tab. 2 confirms), it has two important characteristics that justify its use: to reduce matrix rupture risk and, therefore the delamination of the inner plies; and to form a compact casing with an efficient impact protection.

In the first test using the following set of ply orientations  $[\pm 45^\circ, 90^\circ]$ , the starting stacking sequences (initial population) were chosen. The GA solution was compared with the solution obtained by expert technicians on racing cars design working by intuition and background. Retaining the same stiffness, GA has contributed to a reduction of about 2% of the composite wing weight. With a little loss in stiffness, instead, the reduction increased by about 8%. Additional sets of orientation angles were tested. More significant reductions in weight and stiffness were achieved using the set  $[\pm 30^\circ, 90^\circ]$ .



**Figure 6 Variation of fitness value**

## 6. Conclusion

In the present paper, the possibility of optimizing the stacking sequence of a composite wing by making use of a GA was investigated. It was shown that this powerful non-traditional optimization method could contribute to a considerable reduction of the composite wing weight. The results obtained are encouraging and suggest that further improvements could be obtained investigating different orientation sets. It is also evident that this first implementation requires additional refinement to increase the efficiency and reduce computational time. With minor adjustments the GA presented could be applied successfully to optimize other components of competition cars.

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