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MATHEMATICAL MODELS FOR DETERMINATION OF THE VALUES OF EXTERNAL DYNAMIC INERTIA MOMENTS OF DRIVER'S HAND WHILE OPERATING A VEHICLE

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1. Introduction

Evaluation of steering elements and monitoring equipment in driver's cab can be properly conducted by means of a bio-mechanic analysis of a series of driver's operational motions. In describing motions of driver's segments (arms and legs), there appear dynamic moments of inertia as essential notions to be used in description of motion. They represent the main group of driver's dynamic anthropo-measures, and therefore the knowledge of these values represents a key precondition for solutions of assignments in ergonomic analysis of driver's environment design.

This paper presents a mathematical model for determination of values of external dynamic moments of inertia of driver's arms and legs during vehicle operation with respect to the axes of a coordinate system located at the center of shoulder joint.

For the purpose of defining the stated mathematical model Fig. 1 shows basic spatial model of driver's arm during single phase of driving. The external dynamic moment of inertia for the entire arm, at any observed instant t, can be calculated, according to Steiner's rule, by reducing the arm segments dynamic moments of inertia onto the axes of coordinate system $\{X_0Y_0Z_0\}$ located at the center of shoulder joint. In this case the coordinate system $\{X_0Y_0Z_0\}$ represents a reference frame.

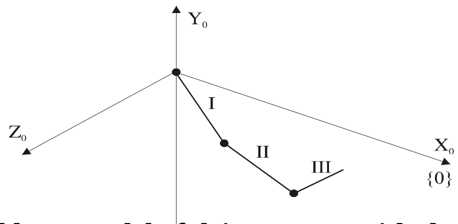

Figure 1. Spatial base model of driver's arm with the model's links

In order to reduce arm segments dynamic moments of inertia onto the reference coordinate system axes it is necessary to know the positions of arm segments center of inertia, as well as the orientation of principal central axes of the segments' inertia within the observed time t, with respect to the axes of the reference coordinate system.

2. Spatial base model of driver's arm

Base arm model (Fig. 1) is composed of a system of rigid bodies connected with joints into a kinematic chain. These rigid bodies represent the base model's links by means of which upper arm, forearm and hand have been modeled. The model's links are connected with joints as defined in biomechanics. The shoulder joint is modeled by way of two cylindrical joints (two degrees of freedom), which suffices for defining spatial position of the arm model. The upper arm and forearm joints are modeled by way of simple cylindrical joint. According to stated kinematics structure the established base arm model represents a complex kinematics chain with at least four degrees of freedom.

The immobile base in respect of which the arm motion is observed has been stated by coordinating system $\{X_oY_oZ_o\}$ located at the center of shoulder joint, and numbered as link 0 (system {0}). The first moving body is the forearm marked as link I, and so on towards the last link of the hand that is marked as link III (Fig. 1).

The arbitrary link presented in Fig. 2 is taken as a rigid body defining the relationship between two neighboring axes of the arm model joints. Joints axes are defined by lines in space or a vector's direction about which link (i) rotates relative to link (i-1).

The model's link is described with two parameters defining relative position between two link axes in space. These parameters are as follows:

a_{i-1} **-** the link length (i-1) measured along mutually perpendicular line of joints axes (i-1) and (i)

α_{i-1} **-** the angle between the axes (i) and (i-1) measured about mutually perpendicular line of joints axes.

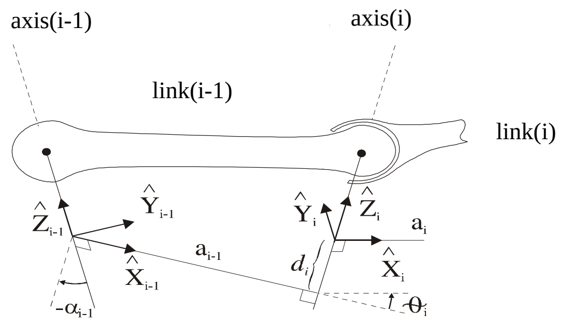

Figure 2. Coordinate systems of model links

2.1 Description of model link connections

The link coordinate systems are rigidly affixed to model links.In Fig. 2 they are indicated as $\{\hat{X}_{i-1}\hat{Y}_{i-1}\hat{Z}_{i-1}\}$ and $\{\hat{X}_i\hat{Y}_i\hat{Z}_i\}$.The notations of links parameters expressed in the link system are as follows:

- a_{i-1} – the distance from $\hat{Z}_{i-1}$ to $\hat{Z}_i$ measured along $\hat{X}_{i-1}$;
- α_{i-1} – the angle between $\hat{Z}_{i-1}$ and $\hat{Z}_i$, measured about $\hat{X}_{i-1}$;
- d_i – the distance from $\hat{X}_{i-1}$ to $\hat{X}_i$ measured along $\hat{Z}_i$;
- q_i – the angle between $\hat{X}_{i-1}$ and $\hat{X}_i$, measured about $\hat{Z}_i$.

For cylindrical (revolute) joint, the joint variables are (θ_i), whereas other variables are the constants. For shoulder joint, modeled with two cylindrical joints, joint variables are (θ_i) and α_{i-1}, whereas other variables equal zero. The introduced notations for link parameters (joint variables (θ_i) and α_{i-1} and joint constants d_i, a_{i-1}) represent Denavit-Hartenberg notation.

2.2 Determination of model links center position and orientation of principal central axes of inertia with respect to reference system axes

The above described base arm model represents a spatial mechanism. For connecting the model's links the Denavit–Hartenberg (D-H) notation was applied. The position of inertia center and the model's links orientation during the motion with respect to reference system have been determined by applying the method of homogenous relative transformations. Using the rules of (D-H) convention in table 1. the spatial position of the arm model is described. The schematic representation of spatial arm model with parameters describing spatial position is presented in Fig. 3. Notations in Fig. 3 are as follows:

- System $\{X_o Y_o Z_0\}$ – the coordinate system according to which motion is observed and the position of model's links defined, marked with $\{0\}$.

Table 1. Kinematic and geometric parameters of three-link spatial arm model

i	*1*	*2*	*3*	*4*
α_{i-1}	*0*	α_1	*0*	*0*
a_{i-1}	*0*	*0*	l_1	l_2
d_{i-1}	*0*	*0*	*0*	*0*
θ_i	$-\theta_1$	*0*	θ_3	θ_4

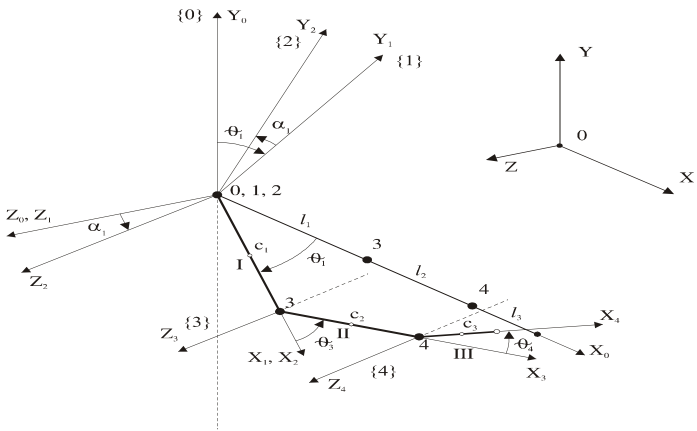

Figure 3. Schematic representation of spatial arm model with parameters describing the arm's spatial position

Systems {1}, {2}, {3} and {4} - are connected to coordinate systems located at joints centers .
By the attached coordinate systems the position and orientation of model links are determined relative to system {0}. (i =1,2,3,4)- joint notation.

- α_1 – the angle between rotating joint axis of joints 1 and 3, measured about $\hat{x}_1$ axis;
- $\theta_1, \theta_3, \theta_4$ – the angles between axes $\hat{x}_{i-1}$ and $\hat{x}_i$ measured about $\hat{z}_i$ axis;
- l_1, l_2, l_3 – the lengths of upper hand, forehand and hand respectively;
- c_1, c_2, c_3 – points of inertia centre of model links.

According to symbolic scheme shown in Fig. 3, the set model represents a cylindrical mechanism with spatial joints with four degrees of freedom. The law of motion for such system can be obtained with knowledge of all (9 + 3+1) reference coordinates. Nine coordinates ($x_{C_i}, y_{C_i}, z_{C_i}$), where i =1, 2, 3 represent coordinates of the links inertia center with respect to zero link of the model, whereas other four coordinates are the angles defining the links orientation and representing generalized coordinates by which 9 reference coordinates can be expressed.

Reference coordinates ($x_{C_i}, y_{C_i}, z_{C_i}$) can be expressed with four generalized coordinates by application of relative transformations properties. The position of vectors of inertia center for all links determined with respect to zero link, at any observed instant, are as follows:

$$\left[{}^0p_{C_1}\right] = \begin{bmatrix} {}^0x_{c_1} \\ {}^0y_{c_1} \\ {}^0z_{c_1} \end{bmatrix} = \begin{bmatrix} c_1 l_{c_1} \\ -s_1 l_{c_1} \\ 0 \end{bmatrix} \qquad \left[{}^0p_{C_2}\right] = \begin{bmatrix} {}^0x_{c2} \\ {}^0y_{c2} \\ {}^0z_{c2} \end{bmatrix} = \begin{bmatrix} (c_1c_3 + s_1s_3c\alpha)l_{c_2} + c_1l_1 \\ (-s_1c_3 + c_1s_3c\alpha)l_{c_2} - s_1l_1 \\ s\alpha s_3 l_{c_2} \end{bmatrix}$$

$$\left[{}^{0}p_{C_3}\right]=\begin{bmatrix} {}^{0}x_{c3} \\ {}^{0}y_{c3} \\ {}^{0}z_{c3} \end{bmatrix}=\begin{bmatrix} \left[(c_1c_{3+}s_1s_3c)c_4+(c_1s_3+s_1c_3c\alpha)s_4\right]*lc_3+(c_1c_3+s_1s_3c\alpha)l_2+c_1l_1 \\ \left[(-s_1c_3+c_1c\alpha s_3)c_4+(s_1s_3+c_1c_3c\alpha)s_4\right]*lc_3-(s_1c_3+c_1c\alpha s_3)l_2-s_1l_1 \\ (s\alpha s_3c_4+s\alpha c_3s_4)*lc_3+(s\alpha s_3l_2) \end{bmatrix} \quad (1).$$

In the equations (1) the following notations are introduced: $s_i=\sin q_i$, $c_i=\cos q_i$, $s\alpha=\sin\alpha_1$, $c\alpha=\cos\alpha_1$

$l_{C_1}, l_{C_2}, l_{C_3}$ - inertia center positions defined in links system.

The orientation of principal central axes inertia of the model links with respect to reference coordinate system axes is described by rotation transformations specified for coordinate systems {2}, {3} and {4}. Rotation transformations are in the following forms:

$${}^{0}_{2}R=\begin{bmatrix} c_1 & s_1c\alpha & -s_1s\alpha & 0 \\ -s_1 & c_1c\alpha & -c_1s\alpha & 0 \\ 0 & s\alpha & c\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad {}^{0}_{3}R=\begin{bmatrix} c_1c_3+s_1s_3c\alpha & -c_1s_3+s_1c_3c\alpha & -s_1s\alpha & 0 \\ -s_1c_3+c_1s_3c\alpha & s_1s_3+c_1c_3c\alpha & -c\alpha s\alpha & 0 \\ s\alpha s_3 & s\alpha c_3 & c\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$${}^{0}_{4}R=\begin{bmatrix} (c_1c_3+s_1s_3c\alpha)c_4+(c_1s_3+s_1c_3c\alpha)s_4 & -(c_1c_3+s_1s_3c\alpha)s_4+(s_1c\alpha c_3-c_1s_3)c_4 & -s_1s\alpha & 0 \\ (-s_1c_3+c_1s_3c\alpha)c_4+(s_1s_3+c_1c_3c\alpha)s_4 & (s_1c_3-c_1s_3c\alpha)s_4+(s_1s_3+c_1c_3c\alpha)c_4 & -c\alpha s\alpha & 0 \\ s\alpha s_3c_4+s\alpha c_3s_4 & -s\alpha s_3s_4+s\alpha c_3c_4 & c\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (2)$$

Given the angles (q_1, q_3, q_4 and a_1) in function of time are known, we can find nine coordinates of inertia center C_i by which concurrently we can obtain the law on inertia center motion for each link of the set. The equations of point C_I coordinates are, in effect, universal cinematic equations of the established arm model.

2.3 Determination of driver's arm segments inertial characteristics

In order to determine driver's arm segments inertia characteristics a method of segments modeling with regular geometric bodies was applied. For this purpose the driver's arm was modeled by way of circular cylinders (upper hand and forehand) , that is an ellipsoid (hand).

From such models, having assumed equal density and homogeneity of matter, by application of known expressions for computing the principal central moments of regular geometric bodies, the principal central moments of inertia for arm segments are calculated.

3. Mathematical models for determination of values of external dynamic moments of inertia

The stated base arm model with installed inertial characteristics of arm segments, as presented in Fig. 3 represents a bio-mechanical model based on which there was stated a mathematical model for determination of the values of external dynamic moments of arm inertia reduced at shoulder joint center.

Rigidly affixed coordinate systems for arm segments are located in the centers of segments masses marked with C_i (i=1,2,3). During the arm motion they change the orientation and position in space. The reference and associated coordinate systems for single position of i^{th} arm segment during vehicle operation are shown in Fig. 4. This Figure also contains the coordinate system $C_i X'_0 Y'_0 Z'_0$ parallel to reference system, to be used for calculation of reduced dynamic moments of inertia of driver's arm.

3.1 Determination of reduced dynamic moments of the model links inertia for reference system axes

Given the origin C_i is the center of inertia of i^{th} link (Fig. 4.), and the axes of coordinate system $C_i X_{C_i} Y_{C_i} Z_{C_i}$, are principal central axes of inertia, then centrifugal inertia moments equal to zero.

Reduced dynamic moment of inertia of i^{th} link for 0 (reference) coordinate system can be defined as follows:

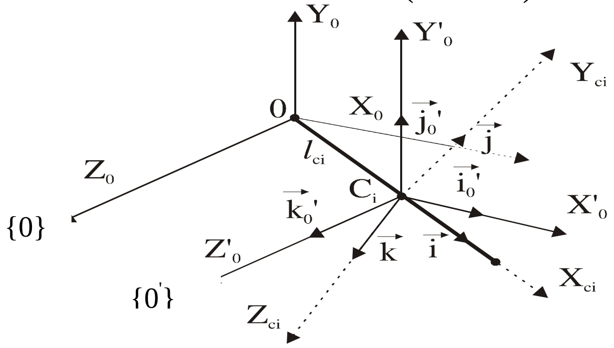

Figure 4. Reference and associated coordinate systems for a single spatial arm position

$$J^{i}_{X_0} = J_{XX_i}\cos^2\angle(\vec{i},\vec{i}'_0) + J_{YY_i}\cos^2\angle(\vec{j},\vec{i}'_0) + J_{ZZ_i}\cos^2\angle(\vec{k},\vec{i}'_0) + m_i(y^2_{C_i} + z^2_{C_i})$$
$$J^{i}_{Y_0} = J_{XX_i}\cos^2\angle(\vec{i},\vec{j}'_0) + J_{YY_i}\cos^2\angle(\vec{j},\vec{j}'_0) + J_{ZZ_i}\cos^2\angle(\vec{k},\vec{j}'_0) + m_i(x^2_{C_i} + z^2_{C_i}) \qquad (3)$$
$$J^{i}_{Z_0} = J_{XX_i}\cos^2\angle(\vec{i},\vec{k}'_0) + J_{YY_i}\cos^2\angle(\vec{j},\vec{k}'_0) + J_{ZZ_i}\cos^2\angle(\vec{k},\vec{k}'_0) + m_i(x^2_{C_i} + y^2_{C_i})$$

Notations in equation (3) are as follows:

$J_{XX_i}, J_{YY_i}, J_{ZZ_i}$ – central inertia moments of the model's links (segments') where i= (1,2,3).

$x_{C_i}, y_{C_i}, z_{C_i}$ – coordinates of the center of i^{th} link inertia mass with respect to reference coordinate. system. These coordinates are determined by position vectors ${}^0\vec{p}_{C_1}, {}^0\vec{p}_{C_2}, {}^0\vec{p}_{C_3}$.

The angles cosines represent projections of unit vectors of $C_i\,X_{C_i}Y_{C_i}Z_{C_i}$ coordinate system, indicated as system $\{C_i\}$, onto the axes of $C_i\,X'_0Y'_0Z'_0$ coordinate system indicated as system $\{0'\}$, where $C_i = C'_i$. The cosines values of the angles between the mentioned coordinate systems are defined by transformation rows of coordinate systems {2}, {3} and {4} with respect to reference system $O\,X_0Y_0Z_0$.

By inserting rotation matrices specified in equations (2) and (1) into the equations (3) dynamic moments of inertia of model links are defined with respect to reference system axes. In these equations central moments of inertia are indicated as follows:

$$A_i = J_{XX_i},\ B_i = J_{YY_i},\ C_i = J_{ZZ_i}.$$

Reduced dynamic moments of inertia of the first link by which the upper arm is modeled, onto the 0 (reference) coordinate system $\{O\,X_0Y_0Z_0\}$ axes are:

$$J^1_{X_0} = A_1c^2{}_1 + B_1(s_1c\alpha)^2 + C_1(-s_1s\alpha)^2 + m_1(-s_1l_{c1})^2,$$
$$J^1_{Y_0} = -A_1s^2{}_1 + B_1(c_1c\alpha)^2 + C_1(-c_1s\alpha)^2 + m_1(c_1l_{c1})^2, \qquad (4)$$
$$J^1_{Z_0} = B_1s\alpha^2 + C_1c\alpha^2 + m_1[(c_1l_{c1})^2 + (-s_1l_{c1})^2].$$

Reduced dynamic moments of inertia of the second link, by which the forearm is modeled, onto the 0 (reference) coordinate system $\{O\,X_0Y_0Z_0\}$ axes are :

$$J^2_{X_0} = A_2(c_1c_3 + s_1s_3c\alpha)^2 + B_2(s_1c_3c\alpha - c_1s_3)^2 + C_2(-s_1s\alpha)^2 + m_2\{[(-s_1c_3 +$$
$$+ c_1c\alpha\,s_3)*l_{c_2} - s_1l_1]^2 + (s\alpha\,s_3l_{c_2})^2\},$$
$$J^2_{Y_0} = A_2(-s_1c_3 + c_1c\alpha\,s_3)^2 + B_2(s_1s_3 + c_1c_3c\alpha)^2 + C_3(-c\alpha\,s\alpha)^2 +$$
$$+ m_2\{[(c_1c_3 + s_1s_3\,c\alpha)*l_{c_2} + c_1l_1]^2 + (-s\alpha\,s_3l_{c_2})^2\},$$
$$J^2_{Z_0} = A_2(s\alpha\,s_3)^2 + B_2(s\alpha\,c_3)^2 + C_2c^2\alpha + m_2\{[(c_1c_3 + s_1c\alpha\,s_3)l_{c_2} + c_1l_1]^2 \qquad (5)$$
$$+ [(-s_1c_3 + c_1c\alpha\,s_3)l_{c_2} - s_1l_1]^2\}$$

Reduced dynamic moments of inertia of the third link, by which the hand is modeled, onto the 0 (reference) coordinate system { O $X_0Y_0Z_0$ } axes are:

$$^{3}_{X_0} = A_3 [(c_1 c_3 + s_1 c a \; s_3) c_4 + c_1 s_3 + s_1 c a \; c_3) s_4]^2 - B_3[(c_1c_3 + s_1 c a \; s_3) s_4 +$$

$$(- c_1 s_3 + s_1 c a \; c_3) c_4]^2 + C_3 (- s_1 s \; a)^2 + m_3 (y^2_{C_3} + z^2_{C3}),$$

$$J^3_{Y_0} = A_3[(-s_1 c_3 + c_1 s_3 c a) c_4 +(s_1 s_3 + c_1 c a \; c_3) s_4]^2 + B_3[(s_1 c_3 - c_1 c a \; s_3)s_4 \qquad (6)$$

$$+(s_1 s_3 + c_1 c a \; c_3) c_4]^2 + C_3 (- c a \; s a)^2 + m_3 (x^2_{C3} + z^2_{C_3}),$$

$$J^3_{Z_0} = A_3 (s a \; s_3 c_4 + s a \; c_3 s_4)^2 + B_3 (-s a \; s_3 s_4) + s a \; c_3 c_4)^2 + C_3 c^2 a \; +$$

$$m_3 (x^2_{C_3} + y^2_{C_3}).$$

Coordinates of inertia centre of the third link are determined by vector position ${}^{0}\vec{p}_{c_3}$ as given in equations (1).
Under application of the rule of summing model links moments of inertia the total reduced moments of the entire arm onto the reference system {O $X_0Y_0Z_0$ } axes are as follows:

$$J_{X_0} = \sum_{i=1}^{3} J^i_{X_0}, \qquad J_{Y_0} = \sum_{i=1}^{3} J^i_{Y_0}, \qquad J_{Z_0} = \sum_{i=1}^{3} J^i_{Z_0} \qquad (7)$$

Equations (7) in expanded form represent a mathematical model according to which, by measuring the angles q_1, q_3, q_4 and a_1 for each position of the stated bio-mechanical arm model in space one can calculate the values of dynamic moments of inertia with respect to immovable coordinate system located at shoulder joint.

4. Conclusions

Based on the established driver's arm model, methods of homogenous transformations and accepted methods of modeling of segments by way of regular geometric bodies, the dynamic moments of inertia have been calculated analytically for the entire arm, having been reduced on coordinate axes with origin in the center of shoulder joint.
The stated mathematical model for spatial motion of arm may be used for computing reduced dynamic moments of inertia for straight (plane-parallel) motion of arm model provided that in the equations (7) for the earlier specified angle α_1 which defines the angle between rotation axes of joints 1 and 3 the assumed quantity is $\alpha_1 = 0$.
Given the laws on orientation angles changes are known: θ_1, θ_3 , θ_4, and α_1 in dependence of time by using the stated mathematical model, one can calculate time changes of dynamic moments of inertia for coordinate system axes located at shoulder joint center.

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