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# SIMULATION OF THE USAGE COVERAGE OF A GIVEN PRODUCT

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## 1. Introduction

In the context of the Usage Coverage Model (UCM) of a product-service, a taxonomy of variables is suggested to setup the link between the design parameters of a product-service and the part of a set of expected usages that may be covered. This paper implements a physics-based model to provide a performance prediction for each usage context that also depends on the user skill. The physics describing the behavior and consequently the performances of a jigsaw is established. Simulating numerically the usage coverage is non trivial for two reasons: the presence of circular references in physical relations and the need to efficiently propagate value sets or domains instead of accurate values. For these two reasons, we modeled the usage coverage issue as a Constraint Satisfaction Problem and we result in the expected service performances and covered usage.

## 2. Presentation of the usage coverage model

We have recently proposed in [Yannou, Chen, Wang, Hoyle, Drayer, Rianantsoa, Alizon, Mathieu, 2009.] a Usage Coverage Model (UCM) so as to get a more thorough marketing model based on sets of permitted usages for a product-service instead of the conventional perceived marketing attributes. In this model, customers are understood as product employers and products as service providers. This method proposes to quantify individuals' performances during product usage, depending on the usage context and on the personal skills of the individuals. It offers the advantage of linking with user experience to introduce the perceived quality of a product's service, as well as to consider particular service delivery conditions. In this way, the UCM model is able to distinguish between the quality (or resulting performance) of the product's service results and the quality of the product's service delivery process (comfort, pleasure, safety).
Let us introduce some notations. Variable set $E$ represents any variables that describe the conditions under which the product is used to provide the service; this is the *usage context*. *X* are product *design parameters*. *Cs* are *user-related parameters* that affect performances, mainly the *skill*, *gender* and *age*. Then one states that individuals' performances $Y$ during product usage depend on the product itself *X*, on the context *E* and on some user-related parameters $C_s$ as depicted by equation (1):.

$$Y = f(X, E, C_s) \tag{1}$$

In fact, A *usage needed* is a set of expected service contexts $E_i$ associated with a usage percentage $F_i$.

$$U_{needed} = \{(E_i, F_i)\} \tag{2}$$

Only a subset of this "usage needed" set may be fulfilled by a given product $X$, this part is called the "feasible usage" and is defined by equation (3):

$$U_{feasible}(X, U_{needed}, C_s) = \left\{ \begin{array}{c} (E_i^*, F_i), \quad such\ that \\ (E_i, F_i) \in U_{needed} \\ \text{and } E_i^* \subseteq E_i \\ \text{and } Y_{r_i} = f_r(X, E_i^*, C_s)\ is\ feasible \end{array} \right\} \quad (3)$$

It means that we look at accomplishing the minimal service so as to be feasible, i.e. effectively cutting a wood board or stick defined by $E_i$ without any other user requirement neither on the quality of the resulting performances – e.g. the $precision$ or the $surface\_rugosity$ -, nor on the preferences on the service processing – e.g., the $linear\_speed$ or the $noise$ -.
Simulating this feasible part of $U_{needed}$ would be much useful to figure out the adequacy of a product with an individual's need.
We propose in this paper to simulate the usage coverage of a product-service. It requires to integrate the product's engineering design parameters in a physics-based model to provide a performance prediction for each usage context. As $U_{needed}$, $Y$ and $U_{feasible}$ represent sets, the computation between these sets requires some special set-based technique. Then, we consider the Usage Coverage Model (UCM) as a Constraint Satisfaction Problem (CSP). We demonstrate on the example of a jigsaw usage that CSP and constraints programming techniques are very convenient for simulating the usage coverage of a product-service.

## 3. Variable screening of a jigsaw for the usage coverage model

### a) Presentation of the jigsaw design issue

We want here to practically apply the Usage Coverage Model to check if a given sized design of a jigsaw matches a given usage need. Its expected usage is "to cut wood dowels and boards of defined materials and dimensions". The jigsaw is visible in Figure 1 and the modeling of its design parameters is provided in Figure 3.


**Figure 1. Jigsaw**

The relations are established between performances *Y* and *X*, *E* and *Cs* through a series of intermediate variables which are mainly here forces, torques and speeds. The Usage Coverage Model of the jigsaw design issue is represented in Figure 2. This figure is a graphical representation of the numerous equations encoding the physics of a jigsaw (more than 30). The different categories of variables are briefly enumerated in Table 1.

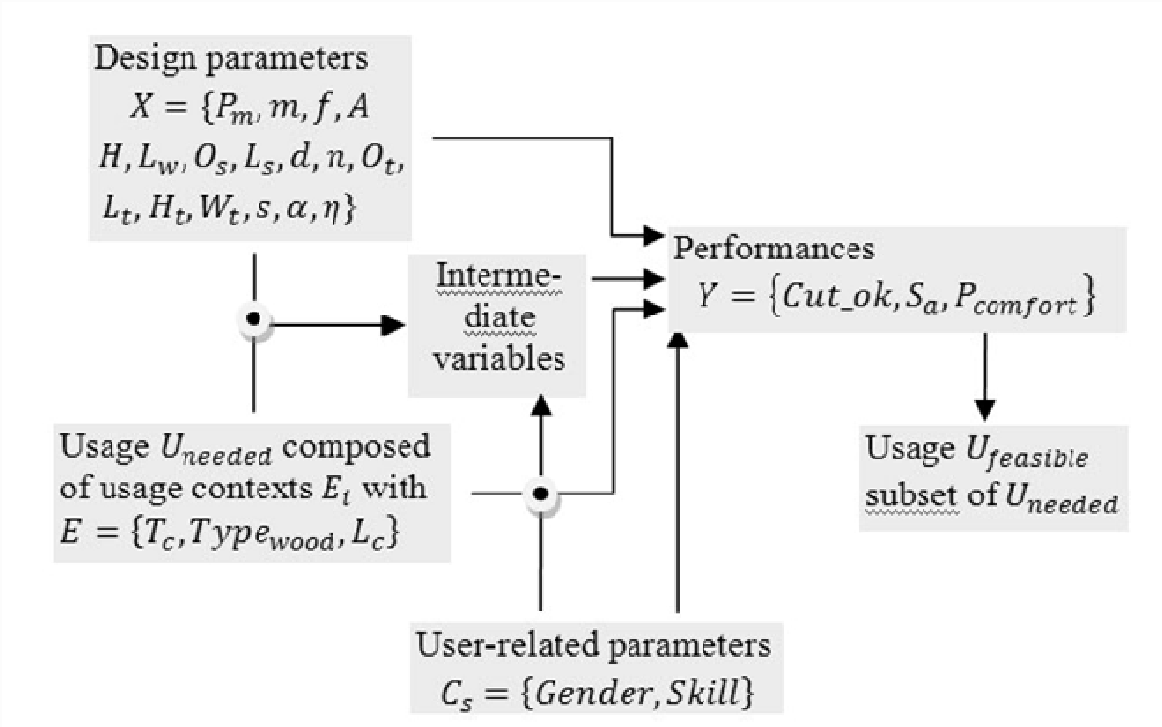


**Figure 2. Variable screening for the jigsaw usage coverage modeling problem**

**Table 1. Enumeration of variables for the Usage Coverage Modeling problem of a jigsaw**

| | |
|---|---|
| USAGE $U_{needed}$ (unique context E) | |
| $T_c$ $(m)$ | *Thickness* of the wood board |
| $Type\_wood \in \{teak, oak \ldots\}$ | Type of wood |
| $L_c$ $(m)$ | *Length* of the wood *cut*. In practice this variable has no influence onto intermediate variables (mainly forces and torques) and performance variables |
| CUSTOMER VARIABLES $C_s$ | |
| $Gender \in \{male, female\}$ | Gender of the saw user |
| $Skill \in \left\{\begin{matrix} Professional, \\ Medium, Basic\ user \end{matrix}\right\}$ | Skill of the user for cutting a piece of wood with a tool of the $X$ type (e.g., a jigsaw) |
| PERFORMANCES $Y$ | |
| $Cut\_ok$ $(boolean)$ | Effectiveness of the cut |
| $S_a$ $(m/s)$ | Mean *advance speed* |
| $P_{comfort}$ (%) | Degree of comfort in the user wrist |
| DESIGN PARAMETERS $X$ | |
| $P_m$ $(W)$ | *Power* of the engine |
| $m$ $(kg)$ | *Mass* of cutting tool |
| $f$ $(round/s)$ | Stroke *frequency* |

| | |
|---|---|
| $A\ (m)$ | Blade translation |
| $H\ (m)$ | Wrist position *height* |
| $L_w\ (m)$ | *Wrist* position *length* |
| $O_s\ (m)$ | *Slider origin* position |
| $L_s\ (m)$ | *Slider length* |
| $d\ (m)$ | *Distance* of the reaction force of the wood onto the slider from the teeth |
| $n\ (no\ unit)$ | Number of teeth |
| $O_t\ (m)$ | *Teeth origin* position |
| $L_t\ (m)$ | *Teeth length* |
| $H_t\ (m)$ | *Teeth height* |
| $W_t\ (m)$ | *Teeth width* |
| $s\ (m)$ | *Step* between two teeth |
| $\alpha\ (°)$ | Rake angle of teeth |
| $\eta\ (°)$ | Clearance angle of teeth |

**b)The user-related parameters Cs**

We consider two demographic variables $C_s = \{Gender, Skill\}$ which are user-related parameters that affect performances. These two variables define the maximal allowable bounds $F_{t\,max}$, $F_{p\,max}$ and $M_{w\,max}$ of, respectively, the translation force, the pressure force and the torque the user wrist may deliver to the jigsaw. These bounds are determined through the *correspondence* table 2; they have been experimentally assessed.

**Table 2. Correspondence table between the gender values and maximal force and torque admissible values**

| Gender | Skill | $F_{t\,max}$ | $F_{p\,max}$ | $M_{w\,max}$ |
|---|---|---|---|---|
| male (70%) | Professional | 100 N | 80 N | 180 N.m |
| | Medium | 80 N | 50 N | 150 N.m |
| | Basic user | 50 N | 20 N | 100 N.m |
| female (30%) | Professional | 80N | 60 N | 150 N.m |
| | Medium | 60 N | 35 N | 120 N.m |
| | Basic user | 35 N | 15 N | 80 .m |

**c)The service performances Y**

We consider three performances for the service of "to cut wood dowels and boards". The most important is the cut effectiveness itself. The second one is the mean advance speed and the third the comfort during the cutting operation.

The cut effectiveness, denoted $Cut\_ok$ is a Boolean which is true as soon as the saw tool is able to get a positive advance speed, i.e.:

$$Cut\_ok = (S_a > 0) \tag{4}$$

The mean advance speed $S_a$ is function of the translation force $F_t$. But it turns out that for hard woods and thick boards, important translation force $F_t$ and engine power $P_m$ are required so as to start the tool advance ($S_a > 0$). A good balance is then required for the cut to be possible.

The comfort of cutting with a jigsaw is mainly due to the wrist torque which must not exceed a maximal amount dependent of the $Gender$ and the $Skill$. It can be expressed by:

$$P_{comfort} = 1 - \left|\frac{M_w}{M_{w\,max}}\right|, \in [0,1] \quad (5)$$

A null value means a null comfort for user when $M_w$ reaches the maximum bearable value $M_{w\,max}$ the user can support.

**d)The service context variables E**

The service context variables are composed of the wood types $Type\_wood$, the thickness $T_c$ and the length $L_c$ of the wood cut.. In practice the length of the wood cut $L_c$ has no influence onto neither an intermediate variable (mainly forces and torques) nor performance variables since forces, comfort and advance speed are intensive or instantaneous measures.

**e)The material relations**

The choice of a $Type\_wood$ implies the wood density $\rho_w$ and the two friction factors $\mu_{sw}$ and $\mu_{tw}$ between, respectively, the jigsaw slider and the wood, and the blade teeth and the wood.
The wood density is dependent from the wood type and is given in the correspondence Table 3[6]. But some uncertainties remain for some wood types depending on the precise essence and origin. In our physics-based simulation system, we deliberately model the densities as value intervals.
According to many engineering sources, the friction coefficients between wood and metal slider on the one side, and blade teeth and wood on the other side generally vary from 0.1 to 0.5. We kept this interval as an epistemic uncertainty (ignorance).

**Table 3. Correspondence table of wood types and densities**

| $Type\_wood$ | Wood density $\rho_w$ estimation | $Type\_wood$ | Wood density $\rho_w$ estimation |
|---|---|---|---|
| Oak | 590 - 930 | Fir | 480.6-608.7 |
| Beech | 736.8 | Plywood | 575-650 |
| Chestnut-tree | 560.6-640.7 | Chipboard / Particleboard | 700-800 |
| Willow | 528.6 | Teak | 630 - 720 |
| Cherry wood | 608.7 | Maple | 576.7-608.7 |
| Pine | 370 - 660 | Alder | 528.6 |

**f)The causal diagram between major variables**

As shown in Figure 4, we suppose that $F_t$, $F_p$ are constant and determined by the nature of the user through the correspondence table 2. Then, the objective is to find out the mean advance speed $S_a$ and the torque $M_w$ in the user's wrist. But:

- to know $F_a$ we should know $F_t$ and $F_f$,
- to know $F_f$ we should know $F_r$,
- to know $F_r$ we should know $F_c$ and $F_p$,
- to know $F_c$ we should know $H_d$ or $F_a$,
- $H_d$ is also related to $F_a$.

We can remark that there is a causal loop between the forces. This phenomenon is highly frequent when dealing with physical systems.

[6] The wood densities have been found on the two following web sites : http://www.gkehe.8m.com/data.htm, www.simetric.co.uk/si_wood.htm.

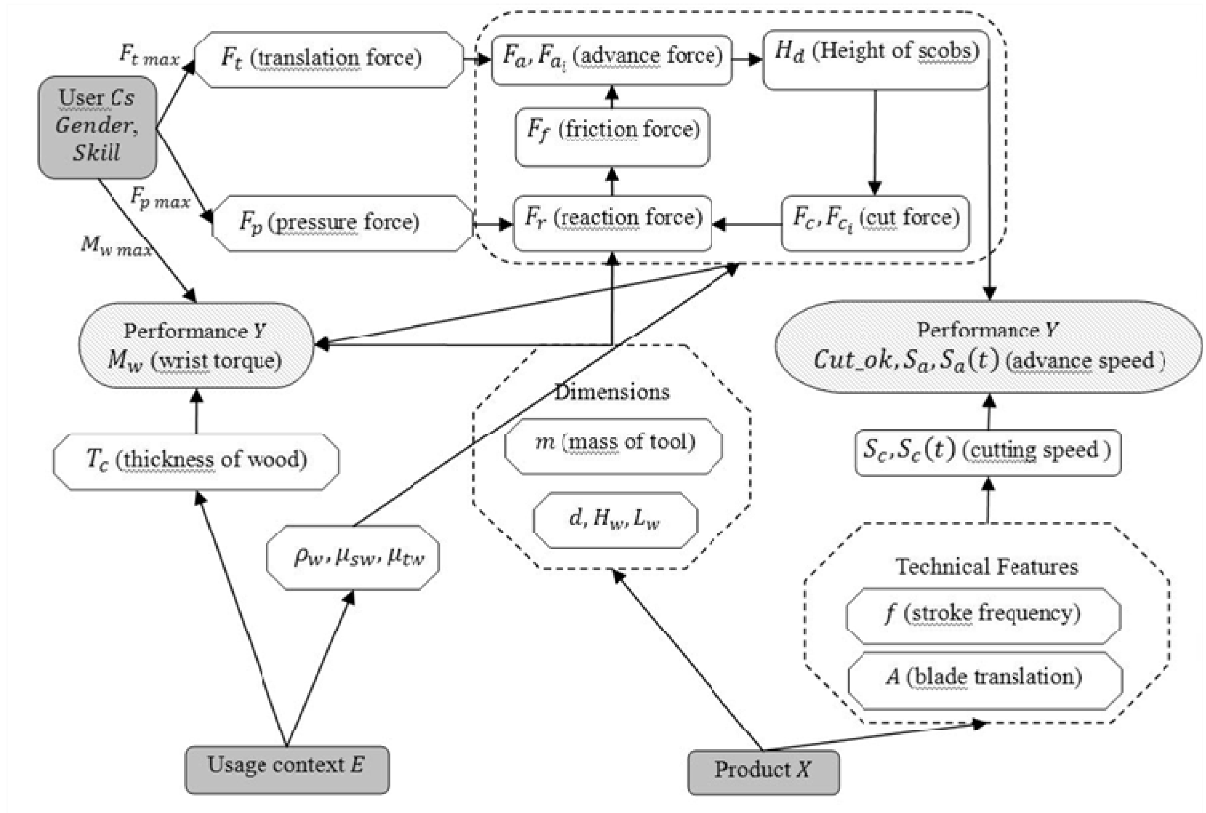


**Figure 4. Causal relations between the variables of the Usage Coverage Modeling issue of a jigsaw**

## 4. Simulations of usage coverage with constraint programming

### a)What is a Constraint Satisfaction Problem (CSP)?

A Constraint Satisfaction Problem CSP [2] is defined by a 3-tuple *(X, D, C)* such that:

- $X = \{x_1, x_2, x_3 \dots, x_n\}$ is a finite set of variables that we call *constraint variables* with *n* the number of variables in the problem to be solved.
- $D = \{d_1, d_2, d_3 \dots, d_n\}$ is a finite set of variable value domains of *X* such that:

$$\forall\, i \in \{1, \dots, n\}, x_i \in d_i \tag{6}$$

A domain should be a real interval or a set of integer values.

- $C = \{c_1, c_2, c_3 \dots, c_p\}$ is a finite set of constraints, *p* being any integer number representing the number of constraints of the problem.

Solving a CSP boils down to instantiating each of the variables of *X* and at the same time satisfying the set of problem constraints *C*.

$$\forall\, i \in \{1, \dots, p\}, \exists\, X_i \subseteq X \,/\, c_i\,(X_i) \tag{7}$$

To do that, CSP solvers use a constraint propagation mechanism as a step by step interval (or domain) reduction process. Over the past years a variety of solving methods have been developed, which enable fast computation of CSP, and supply the user with intervals ensuring to contain all solutions of the CSP; this is the *completeness* property.

A constraint should be any type of mathematical relation (linear, quadratic, non-linear, Boolean…) covering the values of a set of variables. Functions operate on values but constraints operate on domains.

We can find information about propagation techniques and domain reductions in [Tsang, 1993.- Benhamou, Granvilliers, 2006.] for real constraint variables and in [Montanari, 1974- Garrido, Onaindia, Sapona, 2008.] for enumerated and integer constraint variables.

### b)CSP and design problem

During the design process, designers use and manage design rules, tables, abacus, relations… All these structures should be modeled as constraints (mathematical relations between variables).
The CSP community has developed some work applicable to product and system design [Vargas, Saucier, Albert, Yvars, 1994.- Yvars, 2008.]. For instance, dynamic CSPs enable one or more constraints to be added or removed. This allows configuration problems for the management of industrial product options to be processed as in [Aldanondo, Hadj-Hamou, Lamothe, 2003.].

### c)CSP Model for Jigsaw

*Assigning a domain to a design variable*

According to the CSP model we must assign one domain of values to a constrained variable. Table 4 shows the initial intervals allocated, by default, to the different variables for any jigsaw problem.

**Table 4. The default domain assignments for any jigsaw physics-based modeling**

| Variables | Type | Theore-tical domains | First domain assignments |
|---|---|---|---|
| **USAGE U (unique context E)** | | | |
| $T_c\ (m)$ | Continuous | $]0,+\infty[$ | $[0.001, 0.01]$ |
| $L_c\ (m)$ | | | $[0.005, 2.5]$ |
| $Type\_wood \in \{teak, oak \ldots\}$ | Discrete | see correspondence table | |
| $\rho_w\ (kg/m^3)$ | Continuous | $]0,+\infty[$ | $[300, 1000]$ |
| $\mu_{sw}, \mu_{tw} (no\ unit$ | | | $[0.1, 0.5]$ |
| **PERFORMANCES Y** | | | |
| $S_a\ (m/s)$ | Continuous | $]0,+\infty[$ | $[0.001, 0.1]$ |
| $P_{comfort}$ (%) | Continuous | $[0, 1]$ | $[0, 1]$ |
| **DESIGN PARAMETERS X** | | | |
| $P_m\ (W)$ | Continuous | $]0,+\infty[$ | $[50, 3000]$ |
| $m\ (kg)$ | | | $[0.5, 10]$ |
| $f\ (round/s)$ | | | $[1, 500]$ |
| $A\ (m)$ | | | $[0.01, 0.1]$ |
| $H\ (m)$ | | | $[0.05, 0.3]$ |
| $L_w\ (m)$ | | | $[0, 0.2]$ |
| $O_s\ (m)$ | | | $[0, 0.05]$ |
| $L_s\ (m)$ | | | $[0.1, 0.15]$ |
| $d\ (m)$ | | | $[-0.15, 0.05]$ |
| $O_t\ (m)$ | | | $[0, 0.02]$ |
| $L_t\ (m)$ | | | $[0.01, 0.1]$ |
| $H_t\ (m)$ | | | $[0.0001, 0.01]$ |
| $W_t\ (m)$ | | | $[0.0002, 0.003]$ |
| $s\ (m)$ | | | $[0.0005, 0.005]$ |
| $\alpha, \eta\ (°)$ | Continuou | $[0°, 90°]$ | $[0°, 90°]$ |

| | | | |
|---|---|---|---|
| | s | | |
| $n$ $(no\ unit)$ | Discrete | $]1, +\infty[$ | $[3, 40]$ |
| **INTERMEDIATE VARIABLES** | | | |
| $F_t$ $(N)$ | Continuou s | $[0, F_{t\,max}]$ | $[0, F_{t\,max}]$ |
| $F_p$ $(N)$ | Continuou s | $[0, F_{p\,max}]$ | $[0, F_{p\,max}]$ |
| $H_d$ $(m)$ | Continuou s | $]0, +\infty[$ | $]0, 0.005]$ |
| $F_{t\,max}$ $(N)$ | Discrete | see correspondence table | |
| $F_{p\,max}$ $(N)$ | | | |

*Dealing with loops between design relations*

CSP technology efficiently manages cycles in the network of design relations, even in presence of causal loops.

The kind of circularity that has been previously highlighted in Figure 4 cannot be solved with spreadsheets like excel software. CSP solvers manage this loop as a set of constraints. Then all the equations of the Jigsaw problem must be considered as constraints. This is what we did.

*Using a global constraint called constraint table to manage technical tables*

A constraint table is a global constraint that represents the possible combination values of a set of constraint variables. By global constraint, we mean a constraint that should be propagated on complex data structures. In our case, each line of a constraint table is a tuple of consistent values. If one or several values of a constraint variable become forbidden during a CSP solving process all the tuples related to this value are removed from the table too. For example, with Table 2 (Correspondence table between the gender values and maximal force and torque admissible values), if we decide that $M_{w\,max}$ must be greater than 120 N and $F_{p\,max}$ must be different from 50N then, lines number 2, 3, 5 and 6 are removed from the table. Only lines number 1 and 4 stay inside the constraint table as possible options.

## 5. Scenario of the jigsaw usage coverage

Let us imagine that a family is wondering which saw is well adapted to the usage needs of any of its members: the two parents and three teenagers. They have the project to restore a wooden cottage all together. They are more or less skilled with the use of saws. Seven usage contexts for cutting wood have been planned depending on the assigned tasks to the family members (see Table 5). Here, usage contexts are defined by given values of $(type\_wood, thickness)$ like $(\{oak\}, \{0.02\})$ instead of intervals like $(\{oak, plywood\}, [0.015, 0.035])$; the reason being only the simplicity of a first example. Usage contexts are then said *crisp* vectors.

**Table 5. 7 Usage contexts for cutting wood with different users**

| | | Daughter | Mother | Father | Son #1 | | Son #2 | |
|---|---|---|---|---|---|---|---|---|
| | | a | b | c | d | e | f | g |
| Usage contexts $E$ | $type\_wood$ | $oak$ | $oak$ | $plywood$ | $plywood$ | $fir$ | $fir$ | $fir$ |
| | $thickness$ | 0.025 | 0.025 | 0.03 | 0.05 | 0.04 | 0.04 | 0.035 |
| User-related variables $U$ | $gender$ | $female$ | $female$ | $male$ | $male$ | $male$ | $male$ | $male$ |
| | $skill$ | $basic\ user$ | $medium$ | $professional$ | $medium$ | $medium$ | $basic\ user$ | $basic\ user$ |

Their stake is to buy a saw that fulfill at best the different needs, we prefer to say “that cover at best the usages needed”. The jigsaw Bosch PST 50 AE is a candidate they envisage to purchase. The three performances are considered as objectives, no preference constraint is put on them. Table 6 shows the design parameter values corresponding to the Bosch PST 50 AE jigsaw. These data are more constraining than those of Table 4. Let us notice the large value domain for the stroke frequency $f$ between 8.4 and 45.0 $round/s$. Indeed, an electronic variator may address a range of frequencies at the detriment of the cut force $F_c$ for a given fixed engine power $P_m$. This possible variation of $f$ is directly modeled as a value interval in our CSP system.

**Table 6. Design parameters X for the Bosch PST 50 AE jigsaw**

| | | | |
|---|---|---|---|
| $P_m$ $(W)$ | 150 | $d$ $(m)$ | 0.03 |
| $m$ $(kg)$ | 1.5 | $O_t$ $(m)$ | 0.015 |
| $f$ $(round/s)$ | [8.4, 45.0] | $L_t$ $(m)$ | 0.068 |
| $A$ $(m)$ | 0.018 | $H_t$ $(m)$ | 0.002 |
| $H$ $(m)$ | 0.22 | $W_t$ $(m)$ | 0.0012 |
| $L_w$ $(m)$ | 0.09 | $s$ $(m)$ | 0.004 |
| $O_s$ $(m)$ | 0.03 | $\alpha, \eta$ (°) | 18° |
| $L_s$ $(m)$ | 0.13 | $n$ $(no\ unit)$ | 18 |

The simulation results show in Table 7 that with a Bosch PST50 AE jigsaw we can cover the $\{b, c, e, g\}$ subset of the $\{a, b, c, d, e, f, g\}$ initial set of usages. The failure analysis reveals that:

- For usage context $a$: The thickness (2.5 centimeters of $oak$, a notable hard wood) is too important for a $basic\ female$ user.
- For usage context $d$: The thickness is too important for a jigsaw tool.
- For usage context $f$: The cutting operation is impossible for a $basic$ user with this thickness of 4.0 centimeters of $fir$.

For the other feasible usage contexts $\{b, c, e, g\}$, the CSP computation brings the information of the maximal allowable advance speed and the minimal comfort ratio. For instance, for usage contexts $\{b, c, e, g\}$, the advance speed is between $\{1.1, 4.1, 2.2, 1.3\}$ millimeters per second, which is quite a good advance speed. The most tedious operations (advance speed around 1 millimeter per second) are for usage contexts $b$ and $g$, which correspond to non experienced people facing a wood piece of a practical thickness. Usage context $e$ corresponds to a more medium-experienced male and then, the advance speed may reach 2.2 millimeters per second since it directly depends on the maximal forces $F_{t\,max}$ and $F_{p\,max}$ that the user may deliver.

The maximal advance speed of 4.1 millimeters per second is reached for usage context $c$ which corresponds to a $male$ user with a $professional$ skill cutting $plywood$ which is in general less dense than $oak$ (see Table 3). It is not surprising to note that the maximal amount of comfort follows the same order than the advance speed's. We got for usage contexts $\{b, c, e, g\}$, the maximal comfort ratios $\{80\%, 97\%, 91\%, 84\%\}$. The $professional\ male$ is more comfortable in usage context $c$ since his wrist is less sollicitated relatively to the maximal allowable wrist torque.

Another interesting result from the CSP computation is the maximal allowable stroke frequency which is strongly limited to 12 rounds per second, far from the technical possibility of 45 rps. The reason must be that, beyond this value of 12 rounds per second, the translation force $F_t$ applied to the wood section becomes insufficient to get a given positive scob height $H_d$. It denotes a non trivial interaction of physics equations. This notion of minimal translation force $F_t$ is well illustrated by the existence of non-zero lower bound of $F_t$ variable. This phenomenon of a minimal translational force to exert so as to start the advance is easy to notice as soon as the cutting operation is not that easy.

For this first experiment, the Bosch PST 50 AE jigsaw is able to cover 4 usage contexts out of 7. And for these 4 usage contexts, the performance $S_a$ and $P_{comfort}$ are more or less satisfactory.

In a second experiment, we imagine to change the sole design parameter of power of the Bosch PST 50 AE jigsaw, passing from 150 W to 200 W. The same CSP computations are performed (not showed here), leading to an increasing of the usage coverage since usage contexts $\{a, f\}$ are now made feasible.

**Table 7. 7 CSP results for the {a,b,c,d,e,f,g} usage needed set**

| | a | b | c | d | e | f | g |
|---|---|---|---|---|---|---|---|
| $Cut\_ok$ | **0** | **1** | **1** | **0** | **1** | **0** | **1** |
| $S_a\ (m/s)$ | | [0.00100000, **0.00111064**] | [0.001, **0.00405477**] | | [0.00100000, **0.00221003**] | | [0.00100000, **0.00133189**] |
| $P_{comfort}$ | | [**0.779321**, 0.796168] | [**0.811661**, 0.966039] | | [**0.806973**, 0.908946] | | [**0.764855**, 0.835711] |
| $f\ (round/s)$ | | [8.4, **12.31**] | [8.4, **10.4778**] | | [8.4, **9.36761**] | | [8.4, **10.7535**] |
| $F_t\ (N)$ | | [**37.3677**, 60.00] | [**45.9523**, 100] | | [**53.3138**, 80] | | [**44.4207**, 50] |
| $F_{t\,max}\ (N)$ | | 60 | 100 | | 80 | | 50 |
| $F_p\ (N)$ | | [0, 35] | [0, 80] | | [0, 50] | | [0, 20] |
| $F_{p\,max}\ (N)$ | | 35 | 80 | | 50 | | 20 |
| $H_d\ (m)$ | | [0.0000090260, 0.0000248901] | [0.0000106045, 0.0000429988] | | [0.0000118612, 0.0000262136] | | [0.0000103325, 0.0000137619] |
| $\rho_w\ (kg/m^3)$ | | [590.000, 864.640] | [575.000, 650.001] | | [480.600, 535.961] | | [480.600, 589.302] |

The degree of usage coverage seems apparently to evolve from 4/7 to 6/7. But adopting theses numbers to make a decision on the adapted engine power (150 W or 200 W) would be misleading. Indeed, variables defining the usage context may be defined by value domains and we must compare the relative sizes of the final – shrunk – domain and the initial domain. But there exist also an indirect usage-context variable, namely the wood density $\rho_w$, which is a constrained variable due to a stochastic uncertainty about the effective wood density of a given wood type (see Table 3) and which must be taken into account within the measure of usage coverage. Then, we propose the following formula for the computation of the degree of coverage of a single usage:

$$DC_{single-usage} = feasibility \times \frac{\prod_i |usage\_context\_variable\ |_{final} \times \prod_j |indirect\_context\_variable\ |_{final}}{\prod_i |usage\_context\_variable\ |_{initial} \times \prod_j |indirect\_context\_variable\ |_{initial}} \tag{8}$$

It remains, for the jigsaw use case, to:

$$DC_{single-usage} = Cut_{ok} \times \frac{|T_c\ |_{final} \times |L_c\ |_{final} \times |Type\_wood\ |_{final} \times |\rho_w\ |_{final}}{|T_c\ |_{initial} \times |L_c\ |_{initial} \times |Type\_wood\ |_{initial} \times |\rho_w\ |_{initial}} \tag{9}$$

Then, an overall degree of coverage is computed through the formula:

$$DC_{total} = \frac{1}{k} \times \sum_{i=1}^{k} DC_{single-usage\ i} \tag{10}$$

Table 9 provides the values of these degrees of coverage for the 7 usage contexts. We can observe a significant improvement of the degree of usage coverage from 44% to 63% when increasing the engine power. But usage contexts $a$ and $f$ remain hard to fulfill in case of particular dense wood material.

**Table 9. Degrees of usage coverage for {a,b,c,d,e,f,g} usage needed sets for the two jigsaws 150 W and 200 W**

| | a | b | c | d | e | f | g | $\boldsymbol{DC_{total}}$ |
|---|---|---|---|---|---|---|---|---|
| $P = 150\ W$ | 0 | 0.808 | 1 | 0 | 0.432 | 0 | 0.849 | $\mathbf{0.44}$ |
| $P = 200\ W$ | 0.086 | 1 | 1 | 0 | 1 | 0.347 | 1 | $\mathbf{0.63}$ |

# 6. Conclusion

The Usage Coverage Model (UCM) has already proposed in [1] a taxonomy of variables which is convenient to study the link between the design parameters of a product and the usage subset covered by this given product, while considering the skills and preferences of the user.

The present paper has two goals: setting the Usage Coverage Modelling of the jigsaw design issue, and simulating the usage coverage for this design problem with the aid of Constraint Programming techniques.

The first goal has been met. Different clear categories of variable have been defined and numerous relations have been established linking them, sometimes in a circular manner. This non trivial problem cannot be solved by a spreadsheet calculator.

In addition, many variables are initially modelled as value sets like densities, friction factors and stroke frequency. These domains are perfectly taken into account within Constraint Satisfaction Problems. Indeed, value sets are propagated and shrunk down even in presence of circular relations. Finally, it turns out that Constraint Programming techniques are perfectly appropriate to the simulation of the usage coverage of a product-service. We then propose a clear definition of a degree of usage coverage after a CSP simulation.

In further simulations, we will also consider complex usage contexts defined by value domains so as to better test the relevance of our metrics of usage coverage.

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