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MODELLING OF DISPLACEMENT COMPRESSORS USING MATLAB/SIMULINK SOFTWARE

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Abstract

Model based development is increasingly used to minimise costly and time consuming testing of physical prototypes during product development. Modelling and concept evaluation by using computer models for performance prediction will then be a substantial part of the PD process synthesis-analysis loop. However, easy-to-use and flexible methods have to be used for modelling, especially during product concept development, to avoid traditional coding and debugging. In this case, a twin-screw compressor has been modelled using the visual programming language MATLAB/Simulink. One of the objectives was to investigate whether it was feasible and appropriate to use MATLAB/Simulink for such a complex task. New developed methods have been used for this. The main difficulty turned out to be how to create a queue-like environment for the chambers. This problem was eventually solved, but it shows how MATLAB/Simulink can be cumbersome to use when dealing with complex problems.

1 Introduction

The twin-screw compressor was invented in Sweden in 1934, it was then known as the Lysholm compressor. It has been used in many different fields: for compressing air, for refrigerating systems and more. Today the fuel cell industry is interested in this type of compressor due to its ability to produce large pressure ratios at low volume flows (compared with more conventional turbo compressors) [Kulp 2001]. Compressed air is supplied at the cathode in PEM fuel cells, where the oxygen is consumed. By increasing the stack pressure, the power/weight ratio of the fuel cell is increased, and the weight and dimensions can be reduced, which is important in automotive applications. Also, an expander of twin-screw type can be used for recovery of fluid energy into mechanical energy.

When simulating fluid systems, several methods exist. CFD (Computer Fluid Dynamics) is used to calculate 2 or 3 dimensional flow patterns, but it is time consuming, and simulating transient behaviour is also difficult in CFD. Commercial programs like Flowmaster also exists, but they are more specialised not to mention expensive. This paper shows how one dimensional models of a lumped fluid system, having non-steady-state properties can be developed relatively easily using a visual programming environment, allowing rapid evaluation of concepts. A typical application in product development is when different configurations or layouts are to be examined, as well as parametric studies e.g. for sensitivity analysis and optimisation.

2 Research objectives

The main research question is:

- What are the benefits and drawbacks using the Simulink programming language when modelling fluid systems and fluid machinery, and screw compressors in particular?

This question can in turn be divided into these sub questions:

- How accurate are the results compared to using conventional methods?
- How easy is it to set up the simulation model?
- How flexible and adaptable for different system configurations is this modelling/programming approach?

3 Theory

3.1 Simulation based design

Simulation is a part of the *basic design cycle,* as defined by Roozenburg & Eekels [1995]. It is the step between synthesis and evaluation. When developing products, the basic design cycle can be run through several times, for different sub problems and refinement of the design. In a phase model, as defined by Pahl & Beitz [1988], the simulation method developed in this paper can be used during the conceptual design, embodiment design and detail design. For example, in the concept phase, the system layout of fuel cell systems can be investigated using the models derived in this paper. In embodiment and detailed design, more sophisticated models can be made to predict the performance of the component. Even in the first phase, "clarifying the task", simulation can be useful, to give information if the product has any commercial potential and is worth developing.

3.2 Using a visual programming environment

Simulink falls under the definition of being a visual programming environment, since it "allows the user to specify a program in two- (or more) -dimensional fashion" [Myers 1990]. Some of the benefits of using visual programming are immediate visual feedback, explicit depiction of relationships, and the need to learn fewer programming concepts [Marcinak 2002]. Visual programming met a lot of scepticism among programmers in the late 80's when the first languages were introduced [Myers 1990], but now they are getting more and more common, even for writing large complex systems [Marcinak 2002].

For solving systems of differential equations, using the Simulink visual programming environment is easy compared with conventional programming languages, since the program automatically sorts out the integration order and it has built in solution algorithms. It is easy to get started and to get results quickly.

The drawback of using Simulink is that it is not as flexible as an object oriented programming language. For complex models, many lines have to be connected and many complex equations have to be entered, and the overview is somehow lost when too many blocks are interconnected.

3.3 Modelling of screw compressors

Ever since the invention of the twin-screw compressor in 1934, much effort has been made to predict their performance. During the last 20 or so years, computer programs have been developed, taking more and more phenomena in account. The main factor that differ the twin-screw compression process from an ideal adiabatic compression process is internal leakage in clearances between the chambers in the compressor, and different forms of heat exchange [Kauder et al 2002]. The leakage flow depends on the pressure in the chambers; the pressure depends in turn of the leakage flow but also on the heat exchange and the volume of the chamber. Several chambers exist, and their volume and leakage paths depend on the rotational angle. This is in other word a *complex* problem, using the definition from *Webster's Third New International Dictionary* [1961] where complex is defined as "having many varied interrelated parts, patterns, or elements and consequently hard to understand". Kauder et al [2002] suggests a generic method on how rotary displacement machines should be modelled in computer software. This is an object-oriented approach where the configuration of the machine easily can be changed and phenomena like heat exchange can be included or excluded at will.

4 Modelling approach

4.1 Model overview

To model the compressor, some assumptions have been made. In this model only dry air, where the perfect gas law is valid, is used as working medium. Two-phase flow for an oil-flooded or a water-injected compressor can be modelled by replacing these equations by their more complex relationships. In order to get the right cause and effect for this system, the model has been systemised using bond graphs [Ljung & Glad 1991]. The chambers are idealised as flow storage elements, i.e. their compressibility is modelled, but not their pressure drop due to internal viscosity. These flow storage elements are called fluid compliance objects in dynamic system modelling [Doebelin 1998]. Between the chambers, so-called flow resistance blocks are modelled, for the leakage flow. Conservation laws have been set up for mass, energy and momentum.

4.1.1 Chamber model

The chambers are, as mentioned above, modelled as compliance objects. In a fluid compliance object the pressure is a function of the net mass flow [Glad & Ljung 2001]. In this example perfect gas is assumed. These equations can be expanded or replaced to simulate real gases or two phase flow, the important thing is to get the pressure as a function of the net mass flow (and other factors like time, phase angle etc.) using conservation laws.

The conservation law for non-stationary systems [Ekroth & Granryd 1994] is used:

$$\begin{aligned} Q = \int_{\tau I}^{\tau II}\left[\dot{m}\left(h+\frac{w^2}{2}+gz\right)\right]_{out} d\tau - \int_{\tau I}^{\tau II}\left[\dot{m}\left(h+\frac{w^2}{2}+gz\right)\right]_{in} d\tau + \\ +\left[m\left(u+\frac{w^2}{2}+gz\right)\right]_{II} - \left[m\left(u+\frac{w^2}{2}+gz\right)\right]_{I} + E_t \end{aligned} \qquad (1)$$

The process is considered as adiabatic:

$$Q = 0$$

The velocity components and potential energy terms are also neglected in the chambers. When this equation is differentiated, it leads to equation (2):

$$\frac{d(m*u)}{dt} = \Sigma\dot{m}_{in}h_{in} - \Sigma\dot{m}_{out}h_{out} - \dot{E}_t \tag{2}$$

Since it is a perfect gas, the internal energy and enthalpy are functions of the temperature only. The absolute values of these quantities are not used in these formulas, so $T=0$ can be chosen as reference point in equation (3) in order to simplify the expressions.

$$\left\{u = c_v T, h = c_p T, \dot{E}_t = p\frac{dV}{dt}\right\} \tag{3}$$

$$\begin{aligned} &\frac{mc_v dT}{dt} + \frac{c_v T dm}{dt} = \Sigma\dot{m}_{in}c_{pin}T_{in} - \Sigma\dot{m}_{out}c_{pout}T_{out} - p\frac{dV}{dt} \\ &\frac{dT}{dt} = \frac{1}{mc_v}\left(\Sigma\dot{m}_{in}c_{pin}T_{in} - \Sigma\dot{m}_{out}c_{pout}T_{out} - \frac{c_v T dm}{dt} - p\frac{dV}{dt}\right) \end{aligned} \tag{4}$$

The perfect gas law, differentiated by time:

$$\begin{aligned} &pV = mRT \\ &\frac{dp}{dt}V + \frac{dV}{dt}p = R\left(\frac{dm}{dt}T + \frac{dT}{dt}m\right) \end{aligned} \tag{5}$$

Equation (4) and (5) together gives:

$$\begin{aligned} &\frac{dp}{dt} = \frac{R}{V}\left(\frac{dm}{dt}T + \frac{1}{mc_v}\left(\Sigma\dot{m}_{in}c_{pin}T_{in} - \Sigma\dot{m}_{out}c_{pout}T_{out} - \frac{c_v T dm}{dt} - p\frac{dV}{dt}\right)m\right) - \frac{dV}{dt}\frac{p}{V} = \\ &= \frac{R}{V}\left(\frac{1}{c_v}\left(\Sigma\dot{m}_{in}c_{pin}T_{in} - \Sigma\dot{m}_{out}c_{pout}T_{out} - p\frac{dV}{dt}\right) - \frac{dV}{dt}\frac{p}{R}\right) \end{aligned} \tag{6}$$

Since the chamber is assumed to have a uniform temperature, the outgoing temperature equals the temperature in the chamber. It is calculated by equation (4), which can be rewritten as

$$\frac{dT}{dt} = \frac{1}{mc_v}\left(\Sigma\dot{m}_{in}c_{pin}T_{in} - \Sigma\dot{m}_{out}c_{pout}T - \Sigma\dot{m}_{in}c_v T + \Sigma\dot{m}_{out}c_v T - p\frac{dV}{dt}\right) \tag{7}$$

4.1.2 Leakage equations

The leakage paths have been modelled as fluid resistance objects, where the mass flow is a function of the pressure difference. The equation for one dimensional compressible isentropic nozzle flow of a perfect gas has been used [Ekroth & Granryd 1994].

$$\dot{m} = \alpha A \psi \frac{p_0}{\sqrt{RT_0}}$$
$$\Pi = \frac{p}{p_0}$$
$$\psi(\Pi) = \sqrt{\frac{2\kappa}{\kappa - 1} \Pi^{2/\kappa} \left(1 - \Pi^{\frac{\kappa-1}{\kappa}}\right)} \tag{8}$$
$$\Pi_{crit} = \frac{p_{crit}}{p_0} = \left(\frac{2}{\kappa + 1}\right)^{\frac{\kappa}{\kappa-1}}$$

In this equation α is the flow contraction factor, which depends on the geometry of the clearance. Above the critical pressure, the mass flow is constant.

At the inlet and outlet, the pressure drop is estimated with the formula:

$$\Delta p = c_f \frac{\rho w^2}{2} = \left\{ w = \frac{\dot{m}}{\rho A} \right\} = c_f \frac{\rho \dot{m}^2}{2A^2\rho^2} = \left\{ \rho = \frac{p}{RT} \right\} = c_f \frac{RT\dot{m}^2}{2A^2 p}$$
$$\dot{m} = A\sqrt{\frac{2p\Delta p}{c_f RT}} \tag{9}$$

c_f is a friction constant which has to be empirically determined [Ekroth & Granryd 1994].

4.2 Modelling of the compressor.

In overview, the twin-screw compressor works by chambers, transporting and compressing the air. The number of chambers depends on the geometry of the compressor. One important curve is the volume curve, which describes the volume of the working chamber as a function of the rotational angle. The internal compression ratio depends on the volume curve and the opening and closing of the inlet and outlet ports. Between the chambers, different kinds of leakage flow exist. All these geometry curves are generated by another piece of software and imported to our program.

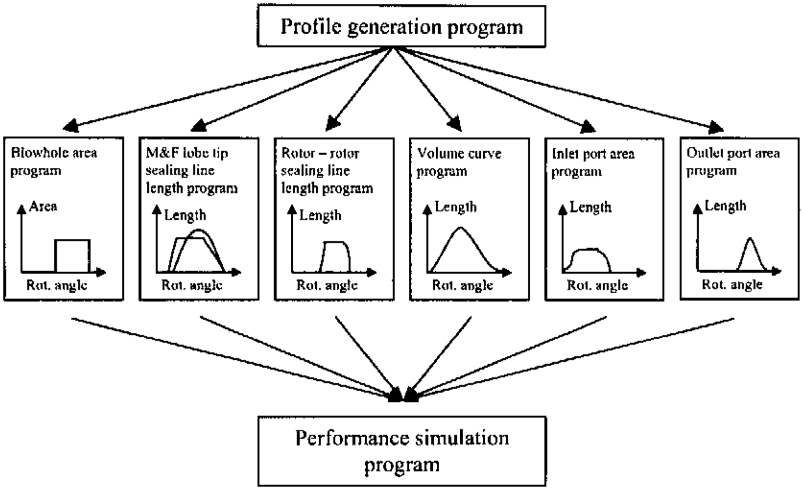

Figure 1: Input data to the performance simulation program [Jonsson 1986]

To describe the modelling approach the volume curve is displayed in Figure 2 and Figure 3 together with the leakage between the chambers. Note that there also exists other leakage paths (see Figure 1), but they are not included in these illustrations, in order to make them more readable. They are however modelled in a similar fashion. The state of the fluid inside a chamber is approximated as homogeneous.

In the first generation of calculation software, one chamber is investigated at a time. For leakage between chambers, the curve is phase shifted and an iteration loop is set up [Jonsson 1986, Platell 1993]. This is illustrated in Figure 2.

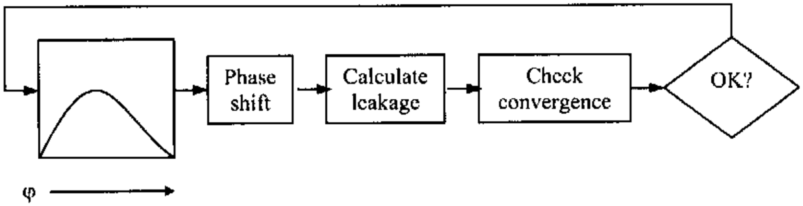

Figure 2. Leakage calculation according to Jonsson [1986] and Platell [1993]

Kauder et al [2002] suggests setting up the chambers as objects in a queue, with leakage paths in between them. A unified phase angle α is used. α is the angle when a new chamber is created. In the screw compressor case, α equals 360° divided by the male lobe number. The chamber vanishes at a certain angle β, when it has completed one cycle (at the end of the volume curve). If β is not a perfect multiple of α, the number of chambers vary by the closest multiple and one more. Example: If $\beta=700°$ and $\alpha=90°$, the number of chambers will be 7 or 8 depending on the rotational angle.

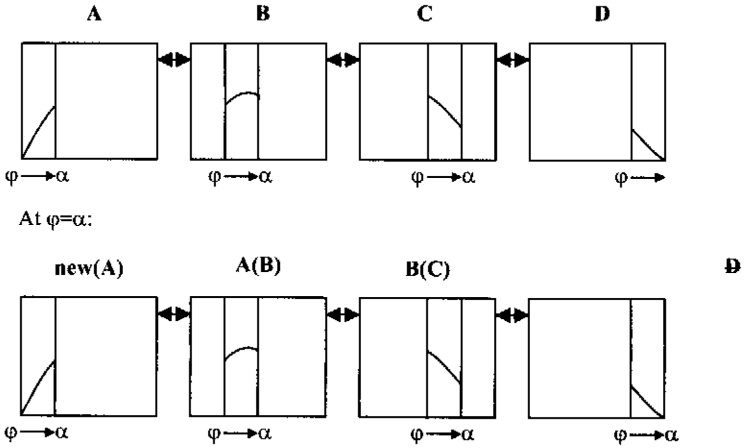

Figure 3. Leakage calculation according to Kauder et al [2002]. This compressor has only four chambers.

In Simulink, the maximum possible chambers (8 in the example above) are distributed with leakage paths in between them. At angles that are multiples of α, all thermodynamic data (pressure, temperature and mass) are transferred one cell ahead. In practice, this is used by applying the MOD-function on the rotational angle to create a saw tooth like function with periodicity α. MOD is the standard mathematical modulus function. This signal triggers the external reset on the integration blocks, and the integration starts over, with external initial conditions from the preceding blocks. The phase angle is phase shifted α, 2α, 3α etc. up to the minimum number of chambers (7α in the example above) and used as input to the following block.

In the last cell, all the data is bypassed at angles where it does not exist. A switch and another modulus function are used to achieve this.

5 Results

At the time of writing this paper, simulations based on real machine data have not been verified so far, due to lack of proper geometry curves (see Figure 1). The model has however been tested using approximate geometry curves, and checked for energy conservation over the working cycle.

6 Conclusion

6.1 Complexity, simplicity and flexibility

One complexity in modelling this kind of problem is to set up a solver algorithm. As mentioned in §3.3, it is a complex problem with many dependencies between the pressures and the flows. The dependencies are also mostly non-linear and time dependent. A possible way to code this in C++, is to set up an iteration loop where the pressures first are calculated, then the flows as a function of the pressures and then iterate until an error tolerance is met. This calculation has to be performed at every time step, and the preceding values have to be used as input to the next time step. The programming of this is indeed a time consuming task. All this is taken care of by Simulink, the dependencies are modelled by drawing lines in between the objects, and then the program automatically solves the problem using its own solution method.

The drawback was the method that had to be used in the modelling of the queue. Instead of adding and removing volumes in the model, as it is done in C++, a workaround solution had to be made. One could say that Simulink is not flexible enough, if flexible is defined as in Nationalencyklopedins Ordbok [2004]: "can (in a convenient way) be adopted to the circumstances". The queue method developed in this paper is working, but it is by no means convenient. A common question when evaluating visual programming languages is if the simplicity of the language is at cost of the flexibility [Myers 1990]. This paper shows that such trade offs can be useful to make when choosing programming environment.

6.2 Reuse of simulations

The equations derived in this paper for the chamber are well suited for generic lumped fluid problems with ideal gases. This could include pipes, filling/emptying of tanks, leakage in clearances, other displacement compressor types and more. The Simulink blocks can be reused with a little modification in such types of simulations.

The queue concept should also be applicable when modelling analogous technical problems in MATLAB/Simulink.

7 Nomenclature

A	cross section area
c_p	specific heat capacity at constant pressure
c_v	specific heat capacity at constant volume
E_t	'shaft work' done by system
g	weight per unit mass
h	specific enthalpy
κ	c_p/c_v
m	mass
p	pressure
p_0	stagnation pressure
ρ	density
R	gas constant $=p/\rho T$
Q	heat added
T	absolute temperature
T_0	stagnation temperature
u	internal energy per unit mass
V	volume
w	velocity
z	height above an arbitrary datum level

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