NordDesign 2008
August 21 – 23, 2008
Tallinn, Estonia

Over- and understeering with a Limited Slip Differential

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Abstract

This paper concerns with the characteristics of the limited slip differential in torque transmission chain. The aim is to compile a vehicle propulsion system model for simulation of vehicle steering process. A model of a new switchable limited slip differential is proposed. The paper presents the energy flow modelling in automotive propulsion systems aimed to provide basic data for the choice of the switchable limited slip differential system parameters in consideration of the impact on the vehicle over- and understeering.

A description of the characteristics and operational conditions of the limited slip differential gear train is given. The basic equations for the description of energy flow from engine to the wheels have been elaborated. Based on these equations, the module system of the vehicle model can be derived. The proposed model makes it possible to observe and guide the distribution of the tracking force during vehicle cornering.

Keywords: modelling, planetary gear train, limited slip, torque transmission, energy loss.

1. Introduction

To improve the safety characteristics of the vehicle its differential mechanism has to control the power flow from the engine to the driving wheels.

The differential gear transfers the torque and rotation to the driving wheels of a vehicle. The main purpose of the differential gear is to provide the wheels with the possibility to rotate at different speeds when driving in curves. The conventional differential gear features a rather simple construction and sufficient reliability. An essential disadvantage appears when one driving wheel slips on ice or some other slippery surface, while one of the wheels rotates at a high speed but the vehicle fails to move. In that situation the conventional differential gear cannot deliver the necessary torque to the other, well-gripping wheel. The limited slip differential can transmit more torque in this case but a decrease in the steering qualities will follow due to the increased understeering.

A lot of constructions of limited slip differential gears are known. A construction of a new switchable limited slip differential gear system is proposed in this paper. The proposed differential gear system will distribute the torque between the wheels more proportionally to the grip of wheels and help the cornering of the vehicle at the same time. It is essential especially for military and off-road vehicles. Cornering at low velocities with no centrifugal forces was investigated.

2. Notation and terms

Main symbol	**Quantity, explanation**	**Unit**
B	track	m
F	force	N
$[J_C]$	constraint Jacobian matrix	[m]
P	power	W
r	radius	m
Δr	force pole offset	m
s	slip	
T	torque	Nm
ΔT	drag or idling torque	Nm
V	translational velocity	m/s
ΔV	constrained velocity	m/s
δ	angle of steering	rad
ω	rotational velocity	rad/s
κ	torque ratio	
ΔF	force difference	
{}	column vector	
[ ]	matrix	

Subscripts

1... *j*	identifier of a shaft
constr	constraint action
i	inner
inert	inertial action
mod	modified (due to losses)
o	outer
pc	special identifier of a planet carrier
pw	special identifier of a planet wheel
sw	special identifier of a sunwheel

Terms:

Differential: a mechanical system with two rotational degrees of freedom, where the gears are mostly arranged as a planetary system.
Limited slip differential: a differential, where the internal relative motion is subjected to torque losses.
Switching process in limited slip differential: a general process of the gear ratio changing in a limited slip differential.

3. Steering process

3.1. Steering in the case of conventional limited slip differential

A vehicle with a limited slip differential is usually understeered. The main characteristic of the limited slip differential is a torque ratio of the differential [7].

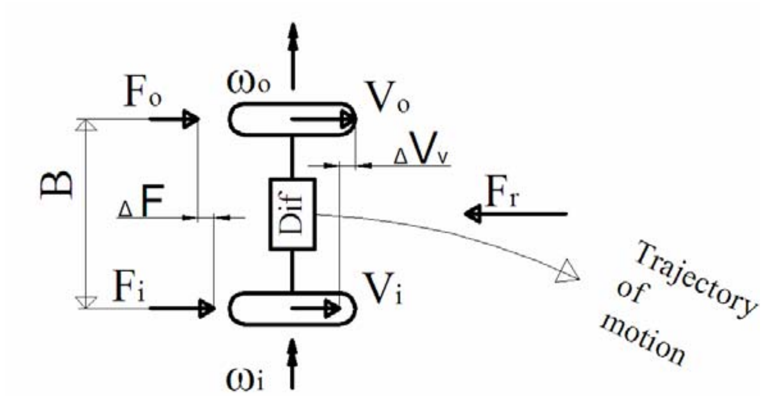

Figure 1. Drive axle. Here B is a track of the vehicle, F - force, V – velocity of the vehicle, the subscripts *o* (outer) and *i* (inner) denote different characteristics on outer and inner wheel in the curve. Subscript *r* denotes the resistance motion of the vehicle.

The torque ratio of the differential can be expressed as $k = \frac{T_2}{T_1}$, where T_2 and T_1 denote the torques of each output shafts of the differential at the moment of the brakes switch-off [7].

To attain a better tractive effort when driving straight forward it is essential to use higher torque ratio. In this case more torque will be transmitted to driving wheels.
As the high torque ratio will disturb the vehicle in cornering, it should be lower when driving into the curve – it will reduce vehicle's understeering. When the vehicle is coming out of the curve the torque ratio should be higher to faster restore the vehicle´s linear motion. To satisfy these terms a change of the torque ratio of the differential is needed.
Instead of changing the torque ratio a similar result could be achieved by changing the differential gear ratio.

3.2. Steering in the case of switchable limited slip differential

The lateral dynamics of a vehicle equipped with a differential of the kind undergoes the following processes.
When the vehicle is moving straight ahead into the curve and steering (front) wheels are starting to turn, it is the stage of a relatively small torque of the driving wheels – the differential is still locked at points 1-2 (Fig. 2). The vehicle is moving along the curve – the rotational velocities of its driving wheels are equal ($\omega_i = \omega_o$). The inner wheel in the curve slips back in relation to the road surface, whereas the outer wheel slips forward in the direction of motion. As a result the tangential force on inner wheel F_i is bigger than on outer wheel F_o and therefore resisting torque $T_o = \Delta F\ 0{,}5B$, which is a reference to the axle of the driving wheels, disturbs vehicle's turning. However, both driving wheels slip back as a result of the motional resistance F_r, see Fig. 1.
At the intermediate stage a relative slip is noted in the friction elements of the differential at points 2 – 3 (Fig. 2). Here we can define the unlocking phase of the differential, which is characterized by the transition from the static friction to the dynamic friction. As a rule, the transition is gradual, depending on the surfaces of friction.

At the stage of the larger angle of the front wheel the differential unlocks at point 3 (Fig. 2). A further increase in the torque of the driving wheels will not bring about an increase in the relative torque ratio. Here at points 3-4 the friction elements are slipping.

After changing the differential gear ratio dynamic friction transforms into static friction again at points 4-5 by locking the differential for a moment. Here actually at points 5-6-7 the friction elements are locked. After a further increasing of the steering angle of the front wheels the differential starts to contribute to turning at point 6 by equating torques on driving wheels and setting them opposite way. Due to the fact that the tangent force on the inner wheel F_i is now less than on the outer wheel F_o, created torque $T_i = \Delta F \times 0{,}5B$ (Fig. 1), a reference to the axle of the driving wheels, helps the vehicle turn. Due to a change of the understeer coefficient, which comes from the mutual change of slip angles [3] of the front and rear wheels, the vehicle moves to oversteering regime and the turning radius will be decreased.

With further increasing the steer angle the friction between friction elements transforms from static into dynamic at points 7-8 and the differential unlocks again allowing slip between its elements.

Between points 8-9 the differential contributes to cornering but after some increase of the steering angle it starts to resist the cornering again. Dynamic friction transforms into static at points 9-10 and the differential locks. The torque value $T_i = 0$ at point 11. The tangent force on the inner wheel F_i is once again bigger than on the outer wheel F_o, and the resisting torque $T_o = \Delta F\ 0{,}5B$ disturbs vehicle's turning.

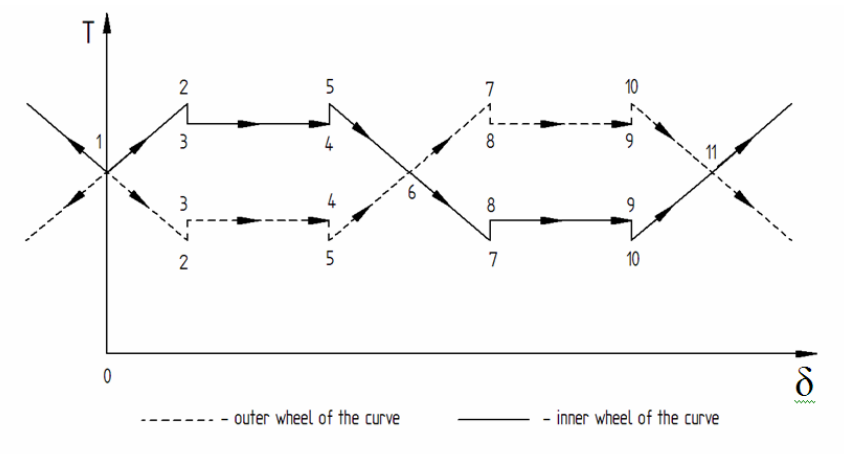

Figure 2. The torque ratio of the differential during the motion along the curvilinear trajectory. Here, T – the driving wheel torque, δ - the angle of the steering wheel, 1, 2, 3 … – characteristic points of the limited slip differential

As a result the differential is expected to provide the vehicle with the best steering qualities allowing thus the maximum power stream from the engine (through transmission) to the wheels.

4. Switchable limited slip differential

4.1. Realisation

In Fig. 3 a new construction of the switchable limited slip differential is shown.

To apply oversteering the driver has to reasonably brake at a run the gears 12, 14 (or gears 8, 10 for opposite turn) (Fig. 3). Reasonable braking means that braking force provides the needed torque ratio value. These stages can be interpreted as modules for the vehicle movement model.

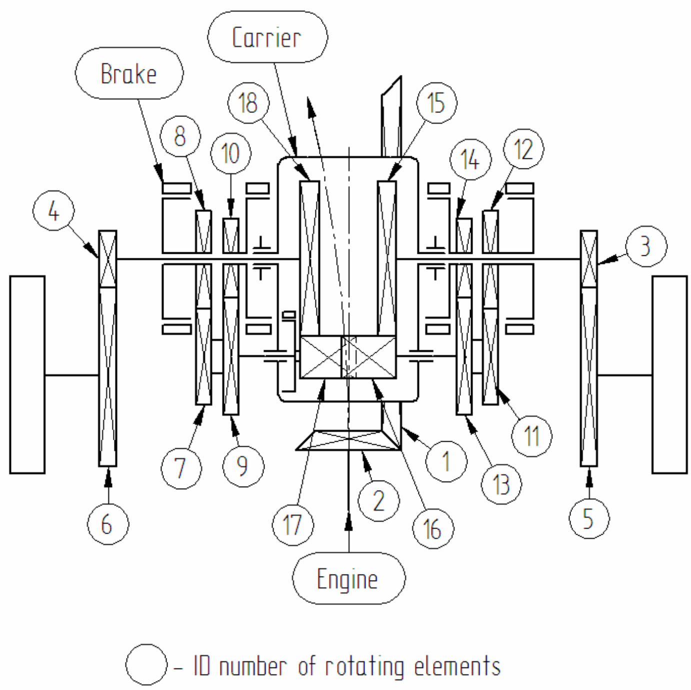

Figure 3. Switchable limited slip differential.
Identification of rotating elements (1…18)

When the vehicle is moving straight ahead or close to that the brake of the gear 17 must be switched on to provide a high torque ratio. Side brakes (gears 8, 12 or 10, 14) will help to achieve the needed turning radius. Side brakes are switching on and off according to the steering wheel turning angle.

During the steering the brake of the gear 17 has to be released to enable the vehicle turning. The first brake (gear 8 or gear 12) helps the vehicle to turn only to a certain turning radius, after that it has to be switched off. The second brake (gear 10 or gear 14) will help to carry on the next part of the turning.

When the vehicle starts to restore the linear trajectory the second brake has to be released and the first brake will help until the vehicle has almost restored its linear trajectory, then the brake has to be released. The brake of gear 17 will be activated again to provide the straight ahead drive.

4.2. Functional model of a switchable limited slip differential

The model has been applied to the system (Fig. 3), that consists of 18 rotating elements and 10 gear meshes (GM), where the planet carrier is in position 1, the sun wheels in positions 18 and 15, the planet wheels in 16 and 17 etc.

System topology: GM1 connects the wheels 1 and 2 (the negative sense for the external gear - here negative); GM2 – 4 and 6; GM3 – 3, 5; GM4 – 17, 18; GM5 – 15, 16; GM6 – 9, 10; GM7 – 13, 14; GM8 – 7, 8; GM9 – 11, 12 and GM10 – 16, 17.
The matrix of the system inherent characteristics focusing on motion constraints is based on relation (1) below. The relationship between the velocities of wheels can be expressed as the virtual differences ΔV in the peripheral velocity of the wheels, which can be set to zero. The motion constraint equations for every gear mesh are as follows:

$$\Delta V = (\omega_m - \omega_{pc})\, r_m \ \pm \ (\omega_n - \omega_{pc})\, r_n = 0, \tag{1}$$

where r denotes the radius of the pitch circle, subscripts m and n are denoting the wheels and pc – the planet carrier. The Jacobian of the internal constraints is:

$$[J_C] = \begin{bmatrix}
-r_1 & r_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & r_4 & 0 & -r_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & r_3 & 0 & -r_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
r_{18}-r_{17} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & r_{17} & -r_{18} \\
r_{15}-r_{16} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -r_{15} & r_{16} & 0 & 0 \\
r_9-r_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -r_9 & r_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
r_{13}-r_{14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -r_{13} & r_{14} & 0 & 0 & 0 & 0 \\
r_7-r_8 & 0 & 0 & 0 & 0 & 0 & -r_7 & r_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
r_{11}-r_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -r_{11} & r_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\
r_{16}-r_{17} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -r_{16} & r_{17} & 0 & 0
\end{bmatrix}, \tag{2}$$

where each row describes the mesh and the columns stand for the rotating elements, respectively.

Compatibility can be written in relations of the velocities, implying zero peripheral speed difference at each gear mesh:

$$\{\Delta V\} = [J_C]\{\omega\} = 0\,. \tag{3}$$

The angular motion velocity vector is then obtained by matrix algebra:

$$\{\omega\} = \begin{bmatrix} [J_C] \\ [J_\omega] \end{bmatrix}^{-1} \begin{Bmatrix} \{0\} \\ \{\omega_{ext}\} \end{Bmatrix}. \tag{4}$$

The external motion constraint Jacobian is the following:

$$J_\omega = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{5}$$

The external motion vector is $\omega_{ext} = [\omega_5\ \omega_6]^T$. Here the superscript T denotes the transpose of the matrix.

The steady state torque equilibrium of loss free motion can be given as:

$$\{T\} + \{T\}_{constr} = \{0\}, \tag{6}$$

where $\{T\}=[I]\{T\}$ and $\{T\}_{constr}$ is the vector of torque contributions from constraint forces given by

$$\{T\}_{constr} = [J_C]^T \{F\}, \qquad (7)$$

where $\{F\}$ is the vector of constraint forces.

The externally applied torques have to be prescribed for all shafts according to current opening conditions:

$$\{T\}_{ext} = [J_T] \{T\}. \qquad (8)$$

The combination of the torque related equations yields the following equation:

$$\begin{Bmatrix} \{T\} \\ \{F\} \end{Bmatrix} = \begin{bmatrix} [I] & [J_C]^T \\ [J_T] & [0] \end{bmatrix}^{-1} \begin{Bmatrix} \{0\} \\ \{T_{ext}\} \end{Bmatrix}, \qquad (9)$$

where $[I\ ]$ and $[0]$ are the identify and zero matrices, respectively; $\{0\}$ – the zero vector; $\{T\}=[T_1.....T_{18}]^T$ is the vector of the externally connected torques and $\{F\}=[F_1....F_{10}]^T$ is the vector of internal constraint (or peripheral tooth contact) forces.

The constitutive relations include also the consideration of losses. The effect of the tooth friction losses can be considered by the modification of the constraint Jacobian: from $[J_C]^T$ to $[J_{Cmod}]^T$.

Then, the internal motion constraints for every gear mesh are as follows:

$$\Delta V = (\omega_m - \omega_{pc})(r_m + \Delta r\, sign(P_{mn})) + (\omega_n - \omega_{pc})(r_n + \Delta r\, sign(P_{mn})), \qquad (10)$$

where r denotes the radius of the pitch circle, subscripts m and n are denoting the wheels and pc – the planet carrier, sign $\pm$ stands for the negative external or positive internal gear mesh and P denotes the power.

Then, the internal motion constraints, which constitute the Jacobian matrix, are as follows:

$$\{\Delta V\} = [J_{C\,\mathrm{mod}}]\{\omega\} = 0 \qquad (11)$$

The equation 7 is valid in non-friction cases. When torque losses are considered, the geometry in terms of pitch radii has to be modified. In the model this is reflected by $[J_{C\,mod}]$. Thus, the relation (7) is transformed into (12):

$$\{T\} = [J_{C\,\mathrm{mod}}]^T \{F\}. \qquad (12)$$

The internal motion constraints, which constitute the Jacobian matrix after modification caused by torque loss, are:

$$J_{C\,\mathrm{mod}\,mn} = (r_m + \Delta r_{mn} sign(P_{mn})) + (r_n + \Delta r_{mn} sign(P_{mn})). \qquad (13)$$

The relative power flow can be determined by the following equation:

$$P_{mn} = J_{C_{mn}} F_m \left(\omega_m - \omega_{pc}\right). \tag{14}$$

The modified torques and constraint forces are as follows:

$$\begin{Bmatrix} \{T_{\mathrm{mod}}\} \\ \{F_{\mathrm{mod}}\} \end{Bmatrix} = \begin{bmatrix} [I] & [J_{C\,\mathrm{mod}}]^T \\ [J_T] & [0] \end{bmatrix}^{-1} \begin{Bmatrix} \{0\} \\ \{T_{ext}\} \end{Bmatrix}, \tag{15}$$

where $\{T_{\mathrm{mod}}\} = T_{\mathrm{mod}_1}, T_{\mathrm{mod}_2}, ..., T_{\mathrm{mod}18}$ – the external torque vector, $\{F_{\mathrm{mod}}\} = F_{\mathrm{mod}1}, F_{\mathrm{mod}2}, ..., F_{\mathrm{mod}10}$ – the constraint force vector of the considered loss.

System torque equilibrium and constitutive equations can be formulated automatically without any special effort.

Conclusion

The paper presents the energy flow modelling in automotive propulsion systems aimed to provide basic data for the choice of the switchable limited slip differential parameters in consideration of the impact on the vehicle over- and understeering.

A description of the characteristics and operational conditions of the switchable limited slip differential gear train is given.

Based on these equations, a module system of the vehicle model can be derived.

The proposed model makes it possible to observe and guide the distribution of the tracking force during vehicle cornering.

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