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TOLERANCE ANALYSIS OF GEOMETRICALLY NON-IDEAL SYSTEMS IN MOTION

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1. Introduction

The success of product development and production processes depends largely upon ensuring at an early stage that the products have the required functional capabilities. The functionality of technical systems in motion is determined by the interaction of their components. This interaction is influenced by small geometrical variations of the components which can originate from manufacturing discrepancies, from deformation due to forces or temperature and from wear and tear between contacting parts in motion. The task of tolerance analysis is to assess the influence of component variations on the functionality of the whole system. The results of this analysis lead to effective tolerance determination and an associated reduction in the manufacturing costs.

All explanations in this paper are exemplified by means of a combustion engine. For this reason, in the next chapter this system is briefly described against the background of geometrical variations and their possible impact on the functionality. After that, tolerance and motion analysis procedures are compared in chapter 3. The task of chapter 4 is to merge these two approaches for an integrated analysis of tolerances and motion. Chapter 5 summarises the paper and points out future prospects.

2. Problem definition for the example of a four stroke combustion engine

The crank mechanism inside a combustion engine controls the thermo-dynamical process. The combustion causes an oscillating movement of the piston which is transformed into a rotating movement of the crankshaft. During the four strokes (intake, compression, working and ejection), the crankshaft performs two rotations; the piston moves twice from the top dead centre to the bottom dead centre and back.

The movements of the engine components are precisely coordinated. The designer has to be aware of the necessary free motion of the parts. High compression ratios and long valve openings in combination with the opposed movement of the piston and the valves lead to comparatively small gaps between these parts [Greuter et al. 2006]. The same problem exists for the piston and the balance weight of the crankshaft. Conflicting design requirements in terms of compact dimensioning (especially the conflict between the dimensions of the balance weights and the dimensions of the piston skirt), without a careful check of the defined tolerances, increases the danger of collisions [Köhler et al. 2006]. The two possible collision scenarios are depicted in figure 1.

As a result of the small gaps between the moving parts, it is important to carefully consider the impact of geometrical variations on the position of each part depending on the crank angle. Geometrical variations can be classified into four groups:

- Dimensional variations (e.g. the distance between the bores of the connecting rod (conrod) is bigger than required)

- Form variations (e.g. the bore is not cylindrical but crowned)
- Location and orientation variations (e.g. the cylinder axis is not located on the principal axis of the mechanism)
- Surface variations

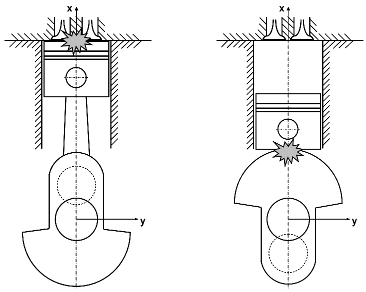


Figure 1. Collision scenarios: piston - cylinder head/valves; piston - balance weight

For the collision of the piston and the balance weight it is important to know that the worst case is not necessarily the bottom dead centre. Depending on the design of the balance weights and the piston skirt, the critical crankshaft position can be situated 20°-40° outside the bottom dead centre [Köhler et al. 2006]. For this reason, it is important to consider the whole course of motion depending on the geometrical variations and not only one crankshaft position (as is usual for tolerance analysis).

3. Tolerance and motion analysis procedures

Both tolerance analysis and motion analysis are based upon the vectorial description of the technical system. Vectors are defined by their tails, their orientations and their lengths. In the two sections of this chapter, tolerance and motion analysis procedures are explained for the combustion engine with the following simplifications: the crank mechanism is considered as planar and the components are rigid bodies.

3.1 Tolerance analysis

The main goal of tolerance analysis is to reveal the impact of defined tolerances for the components on the functionality of the whole system so that an optimal tolerance determination can be achieved. Therefore, the functionality of the system has to be represented by a functional dimension. After that, all relevant individual component dimensions which affect the functional dimension have to be determined. The dimension chain then describes the mathematical relation between the individual dimensions and the functional dimension. Based on the dimension chain, different tolerance analysis methods can be applied to calculate the variation of the functional dimension for varying individual dimensions [Jorden 2004/2005], [Mannewitz 2005], [Klein 2002]. The range of possible variations for the individual dimensions is limited by the defined tolerances. In [Drake 1999] the approach is similar. Vector loops are defined to be able to generate gap/assembly equations (= dimension chain equations) which relate component dimensions to key characteristics (= functional dimensions). The term dimension does not mean that only dimensional tolerances can be analysed. The challenge is to convert the geometrical tolerances to limits for the individual component dimensions in the dimension chain.

In chapter 2, mention was made of the risk of component collisions in the crank mechanism due to component variations. Therefore, tolerance analysis has to be carried out. For the two cases - collision of the piston and the cylinder head/valves as well as collision of the piston and the balance weight - the vector loops are shown in figure 2. The functional dimension is $M_0(\varphi)$. For the collision of the piston and the cylinder head/valves, it is important to get information about the position of the piston head, which depends on the crank angle and the component variations. For the collision of the piston and the balance weight, the position of the piston skirt is important. The individual component variables are

- the crank radius M_1,
- the length of the conrod M_2,
- the compression height M_3 and
- the distance between the piston pin axis and the end of the piston skirt M_4.

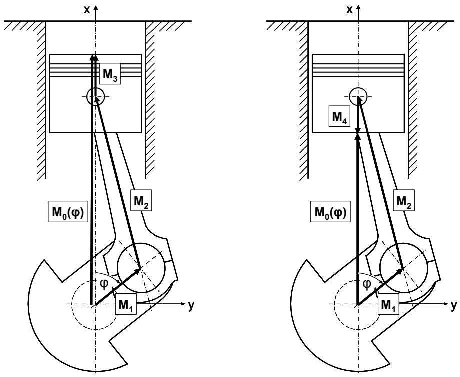


Figure 2. Vector loops for collision analysis

The dimension chain equation for the case on the left hand side can be expressed as follows:

$$M_0(\varphi) = M_1 \cdot \cos\varphi + \sqrt{M_2^{\,2} - M_1^{\,2} \cdot \sin^2\varphi} + M_3 \tag{1}$$

Based on this kind of dimension chain equation, dimensional variations of the mechanism links M_i can easily be handled with arithmetic and statistical tolerance analysis methods for a certain crank angle position (as it is done in [Mannewitz 2005] and [Klein 2002] for an eccentric crank mechanism). However, this dimension chain is limited with regard to variations in form, location and orientation. The location deviation of the cylinder axis, for example, would require an enlargement of the dimension chain equation. In addition to this, the common approach to tolerance analysis does not take into account the movement of the parts depending on the geometrical deviations and, in particular, associated movements between contacting parts due to clearances. This information, however, is required for the analysis of the critical collision points. Due to the fact that movements of the components result from the kinematic and kinetic conditions, it is necessary to combine tolerance analysis with analysis methods for multi-body systems.

3.2 Motion analysis for multi-body systems

Multi-body systems are composed of rigid components which are connected by joints and which move in a certain way due to forces. In the following, the kinematic behaviour of the crank mechanism is described. In addition to this, acting forces at the conrod big end bearing are calculated because in chapter 4, clearance at this joint is assumed.

The bodies of the crank mechanism are the crankshaft (1), the conrod (2), the piston (3) and the crankcase (4) (see figure 3). The bodies are connected by three revolute joints (crankshaft - crankcase, crankshaft - conrod, conrod - piston) and one translational joint (piston - crankcase) at the points O_i and P_i. In the two-dimensional space these joints lead to the following conditions [Haug 1989]:

- O_1 and O_4, P_1 and O_2 as well as P_2 and O_3 coincide
- O_3 moves on the cylinder axis (= x-axis)

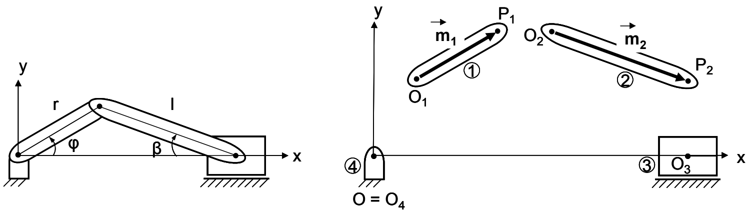


Figure 3. Crankshaft mechanism assembled and disassembled

Only for these conditions can the position of O_3 be defined depending on the crank radius r, the crank angle φ, the length of the conrod l and the conrod ratio λ = r/l with the following equation (which is also part of the dimension chain equation (1)):

$$x(O_3) = r \cdot \cos\varphi + l \cdot \cos\beta = r \cdot \cos\varphi + l \cdot \sqrt{1 - \lambda^2 \cdot \sin^2\varphi} \tag{2}$$

$y(O_3)$ is equal to zero because of the condition that O_3 moves on the cylinder axis. Equation 2 has to be derived twice to get the acceleration of the piston, which is important for the kinetic analysis.
ω = dφ/dt = 2·π·n is the angular velocity which is assumed to be constant.

$$\dot{x} = -r \cdot \omega \cdot \left(\sin\varphi + \frac{\lambda \cdot \sin\varphi \cdot \cos\varphi}{\sqrt{1 - \lambda^2 \cdot \sin^2\varphi}} \right) \tag{3}$$

$$\ddot{x} = a_x = -r \cdot \omega^2 \cdot \left(\cos\varphi + \frac{\lambda \cdot (\cos^2\varphi - \sin^2\varphi)}{\sqrt{1 - \lambda^2 \cdot \sin^2\varphi}} + \frac{\lambda^3 \cdot \sin^2\varphi \cdot \cos^2\varphi}{\sqrt{(1 - \lambda^2 \cdot \sin^2\varphi)^3}} \right) \tag{4}$$

The acting force at the conrod big end bearing $F_{cr,bear}$ can be calculated from:

- the gas force due to the pressure inside the cylinder,
- the translational inertia forces of the piston and the conrod,
- the radial inertia force of the conrod (with $a_r = - r \cdot \omega^2$).

$$F_{cr,bear} = \frac{m_{cr,rot} \cdot a_r \cdot \sin\gamma - F_{rod}}{\cos\eta} \tag{5}$$

with

$$F_{rod} = \frac{F_{gas} + (m_{pi} + m_{cr,osc}) \cdot \ddot{x}}{\cos\beta} \tag{6}$$

$$\eta = \arctan\left(\frac{-m_{cr,rot} \cdot a_r \cdot \cos(90° - \beta - \varphi)}{m_{cr,rot} \cdot a_r \cdot \sin(90° - \beta - \varphi) - F_{rod}} \right) \tag{7}$$

Figure 4 shows the acting forces as well as the polar diagram of the conrod bearing force. The polar diagram gives information about the value and the direction of the bearing force depending on the crank angle (NB: the calculation for the current state in figure 4 yields a negative value for the angle η).

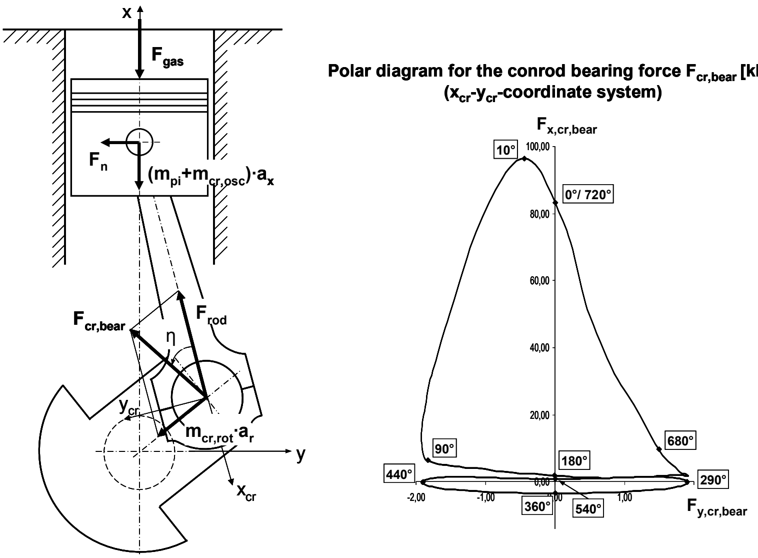


Figure 4. Forces acting at the conrod big end bearing and polar diagram of $F_{cr,bear}$

The relevant parameters for the calculation (deduced from existing diesel engines and standard values for engine calculations) are summed up in table 1. The pressure curve over the crank angle is based on the maximum value and existing pressure curves in established literature.

Table 1. Summary of relevant parameters of the crankshaft mechanism

Parameter and unit	Value
Crank radius r [mm]	45
Conrod length l [mm]	138
Piston diameter D [mm]	84
Piston mass m_{pi} [g]	700
Conrod mass m_{cr} [g]	650
Maximum cylinder pressure p_{max} [bar]	180
Rotational speed n [min^{-1}]	3000

The signs of the conrod bearing force $F_{cr,bear}$ and the angle η are used in chapter 4 for the refinement of the dimension chain equation.

4. Modelling strategy for an integrated analysis of tolerances and motion

The basic step for both tolerance analysis and motion analysis is the same - the definition of the kinematic behaviour. The aim of tolerance analysis is then to calculate, for a certain mechanism position (for the combustion engine that means a certain crank angle), the effects of geometrical deviations on the functionality (that means calculating the variation of M_0 depending on the variations of M_i). In contrast to this, the goal of a motion analysis is to calculate, for fixed geometrical properties, the motion behaviour or the joint forces inside the system.

For the analysis of geometrically non-ideal systems in motion, the basic steps of tolerance analysis procedures (1. definition of the dimension chain, which is equivalent to the definition of the kinematic behaviour, and 2. application of tolerance analysis methods) have to be adhered to. Analysis methods of multi-body systems, especially the calculation of the joint forces, are necessary for the consideration of clearances at joints.

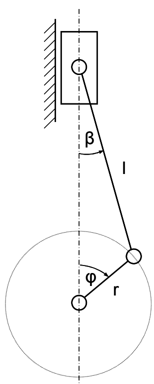
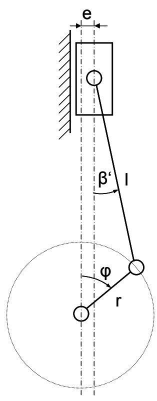
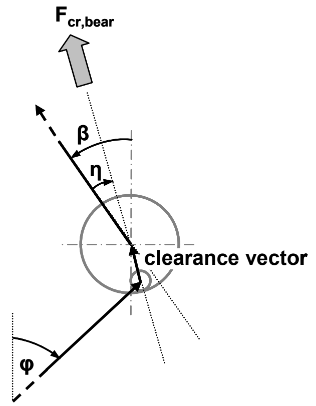
For example, the position of the piston (see equation (2)) is taken into consideration. It shall be shown how different types of deviation affect the piston position during the whole cycle. The following deviations are taken into account:

1. dimensional deviation of the mechanism links crankshaft and conrod
2. position deviation of the cylinder axis
3. clearance at conrod big end bearing.

To facilitate better understanding of the effects, the deviations are considered separately. The basic concept for the modification of the dimension chain as a result of different geometrical deviations is described in [Stuppy 2007]. Dimensional deviations can be handled with the existing dimension chain. The positional deviation of the cylinder axis requires a new definition of the dimension chain equation. Clearances at joints can be taken into account by including a clearance vector in the dimension chain which is already proposed in e.g. [Wittwer et al. 2002] and [Koch et al. 2002]. The important thing is to adjust the clearance vector according to the acting joint force. For the conrod big end bearing, that means that the clearance vector is adjusted according to the bearing force $F_{cr,bear}$. Consequently, information about the sign and the direction of this force (angle η) is required. In comparison to this approach, commercial CAT-systems basically provide the additional option of defining clearances at joints, too. The difference is that there is no possibility to define forces and, consequently, there is no connection between the forces and the direction of the clearance vector. In fact it is the user of the CAT-system who has to define the direction of the clearance vector.

The effects of the three different deviations on the dimension chain equation are shown in table 2 (in the depictions, the deviations have been magnified).

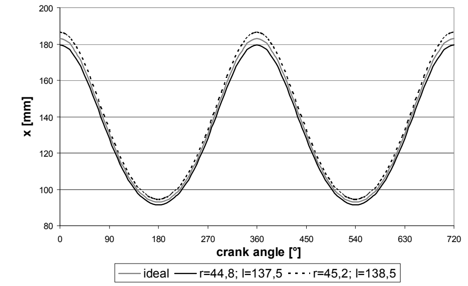
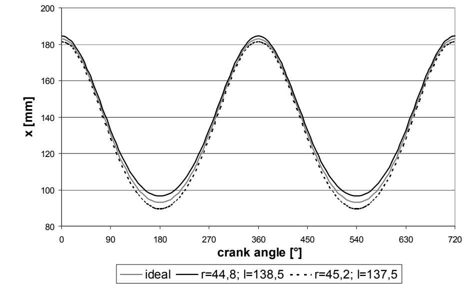
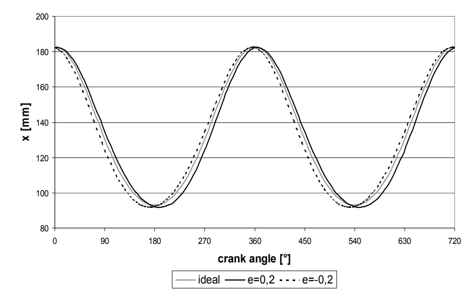
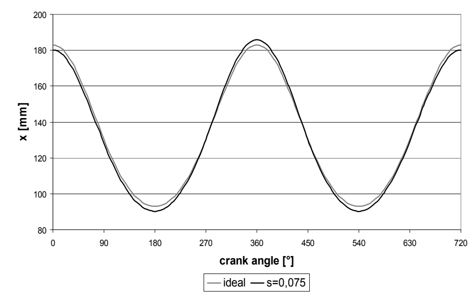
Table 2. Geometrical deviations and their impact on the dimension chain equation

1. Dimensional deviation of mechanism links	2. Deviation of cylinder axis position	3. Clearance at conrod big end bearing
$x(O_3) = r\cdot\cos\varphi + l\cdot\cos\beta$ with $\beta = \arcsin(\lambda\cdot\sin\varphi)$ variation parameters: r, l	$x(O_3) = r\cdot\cos\varphi + l\cdot\cos\beta'$ with $\beta' = \arcsin(\lambda\cdot\sin\varphi - e/l)$ variation parameter: position deviation e	$x(O_3) = r\cdot\cos\varphi + (s/2)\cdot\cos(\beta-\eta) + l\cdot\cos\beta$ with $\beta = \arcsin(\lambda\cdot\sin\varphi)$ and eq. (7) for η variation parameter: clearance s
		

The results for the three types of deviation are shown qualitatively in table 3. In this case, qualitatively means that the curves of the piston position for the non-ideal cases are displayed excessively (a particular magnification factor is specified for each diagram) so that the difference between ideal and non-ideal geometrical properties can be recognized easily. The results have been determined by a worst case analysis where the variation parameters are set to the limit values:

- crank radius r = 45±0,2 mm and conrod length l = 138±0,5 mm
- position deviation of cylinder axis e = ±0,2 mm
- clearance s = 0,075 mm

Table 3. Results

1a. Piston position against crank angle for varying link lengths (magnification factor 5) **I) min r and min l II) max r and max l**	**1b. Piston position against crank angle for varying link lengths (magnification factor 5)** **I) min r and max l II) max r and min l**
	
2. Piston position against crank angle for position deviation of cylinder axis (magnification factor 75)	**3. Piston position against crank angle for clearance at conrod big end bearing (magnification factor 75)**
	

In the first diagram, the piston position is depicted for the condition that the crank radius and the conrod length are both set to the maximum or to the minimum possible values. It can be seen that the deviations affect the top dead centre more than the bottom dead centre. The explanation for this is that the deviations at the top dead centre position are added together, whereas at the bottom dead centre they offset one another due to the varying relative positions of the crankshaft and conrod. In the second case (diagram 1b), the first parameter is initially set to the minimum and the second to the maximum value, and then the other way round. From the diagram it can be observed that, for this configuration, the piston position at the bottom dead centre position is more affected by the deviations. The deviation of the cylinder axis position leads to a change in the extremal values themselves and to a change in their locations (see diagram 2). The extremal values of the non-ideal case are always smaller than the values for the ideal case. In addition to this, the extremal values are no longer exactly at 0°, 180°, 360° etc., but shifted by a few degrees. The differences for the extremal values and their locations at the bottom dead centre position are bigger than those at the top dead centre.

The clearance at the conrod big end bearing affects the piston position in accordance with the forces acting at that joint. For most of the crank angle positions, the bearing force $F_{cr,bear}$ leads to a contact point between conrod and crankshaft throw in the upper half of the bearing. The values of the piston position are then smaller than the ideal values. Only for the range 290°-440° is the contact point in the

lower half of the bearing (this situation is displayed in table 2 point 3). The piston position values for the non-ideal case for this crank angle range are bigger than the values for the ideal case. It has to be taken into account that due to the additional clearance vector in the dimension chain, the kinematic properties of the system are marginally changed. On the basis of the kinematic properties, however, the joint forces are calculated. The effects of this retroaction have to be checked.
From the magnification factors for the diagrams, it can be seen that the influence of different deviations is not always the same. Consequently, the next step is to combine the different kinds of deviation into one model so as to be able to handle different deviations together and, in addition, to be able to identify the main contributors to the result. Moreover, different tolerance analysis methods should be used. In particular, statistical methods should facilitate the integration of production information about the geometrical properties.

5. Summary and future work

In this paper, a modelling strategy for geometrically non-ideal systems in motion is presented. Basic analysis for different kinds of deviation is carried out for the example of a crank mechanism of a combustion engine. With the results gained, understanding of the correlation between defined tolerances and the functionality of the mechanism can be improved. As the vectorial description of the behaviour can be used for all technical systems in motion, the modelling strategy is not limited to the given example, but can also be applied to the analysis of other systems.
The next task is to further develop and refine the model (as already described at the end of chapter 4). The long-term objective of this model is to additionally take into account hydrodynamic effects inside the bearing, deformation of the components due to forces and geometrical changes due to wear and tear.

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