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ASSEMBLY SYNTHESIS FOR OPTIMAL IN-PROCESS DIMENSIONAL ADJUSTABILITY BASED ON A JOINT LIBRARY

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Abstract

Achieving the dimensional integrity for a complex structural assembly is a demanding work due to the manufacturing variations of parts and the tolerance relationship between them. One way to resolve this problem is fabricating all the parts with tight tolerances, which is not an economical way. Another way, which is preferred, is taking advantage of small motions that joints allow such that critical dimensions can be adjusted during assembly operations. This paper addresses this problem by providing a systematic method that decomposes the product geometry at the early stage of design, configures joints and generates subassembly partitioning in such a way that the critical dimensions can be adjusted in assembly processes. The method employs Genetic Algorithm (GA) which generates synthesized assemblies based on a specific joint library for the application. Each synthesized assembly given by the GA undergoes subsidiary optimization routine where the subassembly partitioning is decided for the optimal in-process adjustability by recursively applying a partitioning policy, which is transformed to the well-known minimum cut problem. In order to prove the effectiveness, the method is applied to a three-dimensional automotive space frame example with the accompanying joint library.

Keywords: Design for Assembly, design optimization, computer-aided design, assembly synthesis

1. Introduction

Body frames of most mechanical products such as ships, airplanes, and automotives are fairly complex, hence it is very expensive to manufacture them from a single piece of material if it is not impossible. Typically, human designers would decompose a complex body structure into parts such as panels and beams so that each part could be manufactured with reasonable cost while satisfying its structural and functional requirements.

As the number of parts increases, however, achieving the dimensional integrity of the final assembly becomes more demanding work due to the inherent manufacturing variations in fabrication and assembly operations. For body structures or frames in which parts are typically forged or bent, it is not economical to manufacture every part with tight tolerance such that tolerance stack-up could be compatible with required dimensional integrity of the final product. Hence, in this type of assemblies, while relative dimensions among parts are specified, the locations of joints are not specified at the part design. Instead, during assembly operations, parts are located and fully constrained in fixtures and they are welded or stamped or drilled for fasteners. In order to adjust relative locations, the contact areas or joints should be designed in such a way that a small amount of relative motion is allowed, which is why those contact areas are called slip planes.

To make things more difficult, the assembly sequence also affects achievement of critical dimensions as shown in Figure 1 (modified from [1]). In Figure 1 (a) part 2 and 3 are assembled first and then part 1 is put together. However, when part 1 is attached, there is no slip plane parallel to the critical dimension to absorb manufacturing variations that part {2,3} and {1} might carry. On the other hand, the sequence shown in Figure 1 (b) provides the slip plane at the moment the critical dimension is achieved, so that the slip plane can absorb any variation in length. The Figure 2 (modified from [1]) shows another aspect of interrelation between in-process adjustability and assembly sequence. While the assembly sequence shown in Figure 2 (a) realizes both critical dimensions at the second operation, the sequence shown in Figure 2 (b) achieves one KC at a time with one slip plane or adjustability for each critical dimension, which gives assembly operations better controls. Provided that the geometry of the product is fairly complex with a number of critical dimensions to achieve, decomposing the whole piece into parts, configuring slip planes, defining datums, assigning/analyzing tolerances and planning assembly operations would be a very tedious process and require several iterations. This problem motivated us to develop a systematic method which decompose, configure joints and sequence assembly operations in such a manner that provides critical dimensions with in-process adjustability.

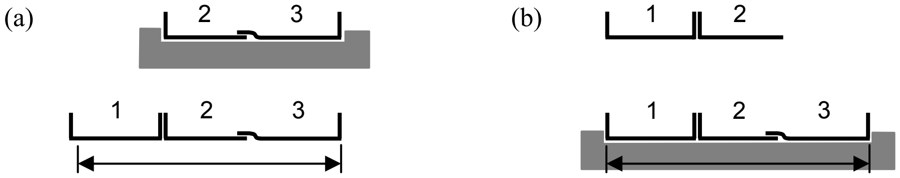

Figure 1. Two assembly sequences for a car floor pan design [1] where total length is critical. In (b), the total length is controllable while it is not in (a).

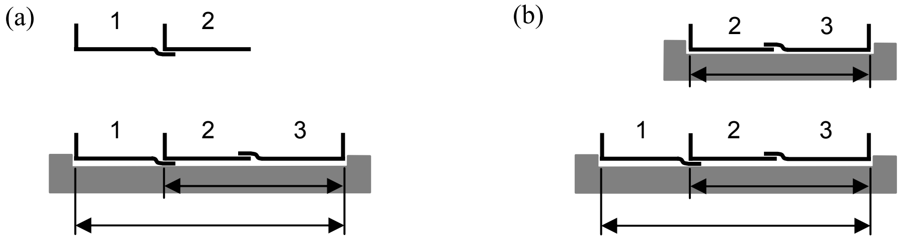

Figure 2. Two assembly sequences [1] where two critical dimensions are overlapped. In (b), each critical dimension is achieved at a time with one slip plane while it is not in (a).

In our previous work [2], we have presented a generative method of assembly synthesis focused on the in-process adjustability and non-forced fit. By recursively decomposing a product from its final shape into parts and assigns joint configurations according to simple rules, the method exploits and represents all possible assembly syntheses with the corresponding assembly sequences which, in combination, achieve dimensional adjustability for critical dimensions and non-forced fit between parts. An augmented AND/OR graph (based on the AND/OR graph by Homem de Mello and Sanderson [3]) has been devised by authors to represent the result. In this work as an extension to our previous work, while we still focus on the assembly synthesis for in-process dimensional adjustability, we utilize the Genetic Algorithm (GA) instead of the generative approach using the AND/OR graph.

The need for using a heuristic optimization method comes from two reasons, one of which is the introduction of joint library. Although we have assigned any arbitrary angles as necessary to provide required adjustability in the previous work [2], in practice, we usually have only a few options in choosing joint types and its orientation. In automotive bodies for instance, a joint configuration assumes the form of lap, butt or lap-butt joint. Hence, the assembly synthesis process should choose only one of these three joint configurations to achieve adjustability, which, depending on the type and the orientation of the joint chosen, can results in slip planes that are not completely parallel to the required adjustability. As a result, it is possible to have one assembly synthesis better than another in terms of adjustability. Another reason for using the optimization method is that the integration of assembly synthesis methods with different objectives such as structural stiffness is expected. As the computation of structural stiffness through FEM takes significant amount of time compared to that of adjustability, the exhaustive search could be inefficient.

2. Nomenclature

- The *Key Characteristic* (KC) has been defined by Lee and Thornton [4] as product features, manufacturing process parameters, and assembly features that significantly affect a product's performance, function, and form. The critical dimensions mentioned earlier will be referred to as KCs in the rest of the paper.
- The "graphe de liaisons fonctionelles" [5] is a simple graph devised to represent an assembly. The graph has a node for each part in the assembly and an edge for each physical contact that a pair of parts have between them. We shall call it the *liaison graph* in this paper, after De Fazio and Whitney [6]. The liaison graph of a synthesized assembly can be defined as a three-tuple, $L_0 = (V_0, E_0, A_0)$, where V_0 is the set of nodes representing parts, E_0 is the set of edges representing joints and A_0 is the set of edges representing KCs.
- A *member* is defined as any section of a product geometry which is allowed to be a separate part after the decomposition process. We shall state a pair of members is connected when they meet at a certain point in the product geometry.
- A *configuration* is defined as a group of members which are connected to at least one of members within the group. A product geometry is simply an initial configuration to be synthesized to an assembly.
- A *decomposition* is a state transition of a configuration into two or more subconfigurations by removing connections.
- A *joint library* is a set of joint configurations available for a specific application. For example, in typical two-dimensional sheet metal assemblies, lap, butt and lap-butt joint would form a joint library.
- A *(synthesized) assembly* is a set of fully decomposed configurations and joints which connect every configuration in the set to at least one of other configurations in the set.
- *Subassembly partitioning* is defined as (the process of building) a binary tree which shows a (partial) assembly sequence of a synthesized assembly.

3. Approach

3.1 Genetic algorithm and joint library

As briefly discussed in the introduction, in this paper, we present an assembly synthesis method for in-process adjustability using GA. Figure 3 depicts how GA interacts with other elements in the system. Inputs for the GA are a product geometry with KCs and a joint library available for the product. With these inputs, at first, the GA initiates candidate solutions – randomly synthesized assemblies, which are then evaluated in terms of in-process adjustability using subsidiary optimal subassembly partitioning scheme devised for this problem. After reproduction routine for specified generations, the GA returns the synthesized assembly with optimal in-process adjustability with its subassembly partitioning plan.

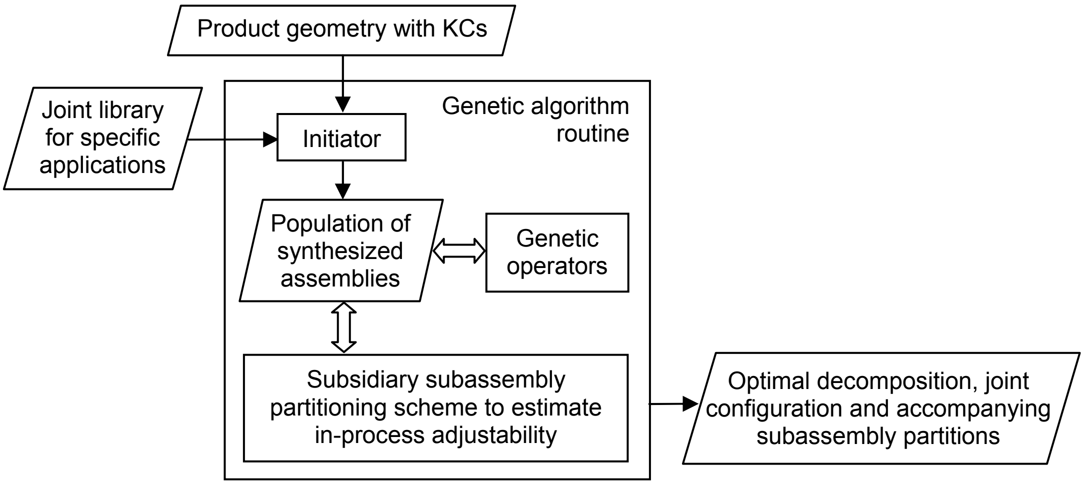

Figure 3. Overview of the presented method.

The initiator within the GA routine generates a specified population of assemblies synthesized by randomly choosing a decomposition and a joint configuration for every connection point. Suppose we have a joint library consisting of lap, butt and lap-butt joints for two-dimensional sheet metal assemblies such as one in Figure 4. Based on the joint library and the product geometry in Figure 5 (a), for example, the GA could synthesize an assembly shown in the Figure 5 (b), which can be represented as a liaison graph depicted in Figure 5 (c).

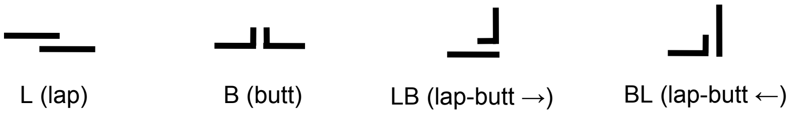

Figure 4. A typical joint library for two-dimensional sheet metal products.

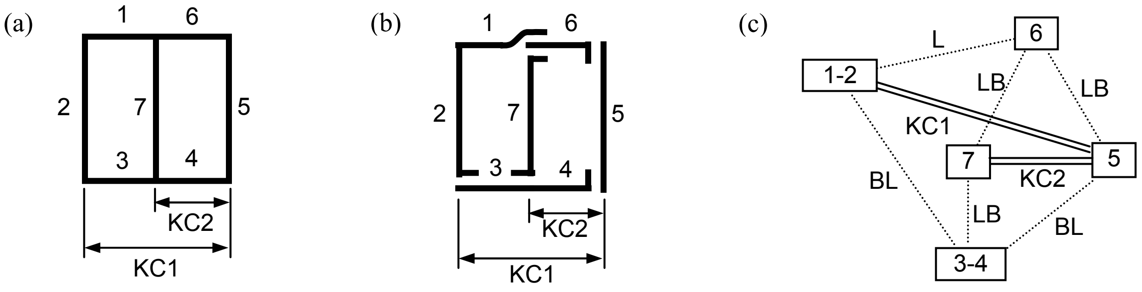

Figure 5. For a given product geometry (a), the GA randomly chooses joint configuration from Figure 4 for each connection point, which results in a synthesized assembly (b). The liaison graph of (b) is shown in (c).

3.2 Subassembly partitioning

Once the GA has obtained a population of synthesized assemblies, these assemblies need to be evaluated in term of adjustability. Because the in-process adjustability is dependent on the assembly sequence as pointed out in the introduction, the system should simulate assembly sequences for evaluation. The system adopts the basic idea of the top-down binary decomposition approach [4] which partitions the final assembly into a pair subassemblies recursively until every subassembly is partitioned to part. However, since we are interested only in in-process adjustability, we do not have to take a look at a full assembly sequence which shows the precedence conditions between all assembly operations which do not necessarily requires adjustability for a KC. Instead, partitioning could be stopped once it reaches a point where no KC is left. In this way, we can reduce the time to compute in-process adjustability of a given assembly, which will eventually make the GA run faster.

3.3 Partitioning rule

As we have observed in Figure 1, it is desirable that a slip plane is provided at the very assembly operation where KC is realized. In Figure 1 (b), no matter what joint configuration the part 1 and 2 have between them, it is important that joint configuration between {1, 2} and 3 should be parallel to the KC's direction. This can be stated in the reverse course as follows: no matter how a subassembly is partitioned further, when a KC is broken by partitioning, slip planes should be oriented parallel to the KC's direction at all the broken connections. However, if partitioning cuts several KCs in different directions, making every joint parallel to every KC is obviously impossible. Therefore, we should not allow partitioning which cuts more than one KC. This constraint should be kept even if KCs are in the same direction because achieving more than one KC at a single assembly operation always requires a compromise as pointed out in Figure 2.

Even if we cut only one KC at a time, we can not always achieve ideal adjustability. Due to the discrete property of the joints available from the library, it is possible that not all the joints being cut are perfectly parallel to the KC being cut by partitioning. Hence we will have to find the cut-set[1] such that the joints' combined parallelism to the KC can be maximized. The parallelism or adjustability of the cut-set to the KC, thus, can be represented as the summation of the inner product of vectors representing the KC and the direction of adjustability of every joint within the cut-set. Since the partitioning at the lower level is dependent of that of the higher level throughout the subassembly partitioning, our objective would be finding the sequence of partitioning such that the summation of the adjustability of cut-sets could be maximized.

One way to search the optimal subassembly partitioning is building and searching the the AND/OR graph which shows all feasible binary trees. Building the AND/OR graph, however, requires considerable computation if the assembly is fairly complex, because all the cut-sets of the liaison graph should enumerated at every partitioning. Moreover, the optimal subassembly partitioning problem is the subroutine of the GA that has many chromosomes to evaluate for many generations. In order to facilitate the GA in searching the optimal subassembly partitioning, we have chosen a dynamic partitioning approach which takes the cut-set of maximum adjustability at a given subassembly and records this partitioning in the binary tree until no node in the BT has a KC left to cut. Since every partitioning is supposed to cut only one KC, if we can obtain the optimal cut-set for each KC, we can compare the

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[1] A *cut-set* in a connected graph $G = (V, E)$, is a minimal set of edges of E whose removal from G, renders G disconnected [7]. In the liaison graph, we do not count edges representing KC to a cut-set.

adjustability of the optimal cut-set of one KC with another so that we can choose the best of the optimal cut-sets for all KCs. For a KC of interest, say a_1, we assign $(1-|\mathbf{k}(a_1)\bullet\mathbf{n}(e)|)$ to all edges, where $\mathbf{k}(a_1)$ is the vector of the adjustability for the KC and $\mathbf{n}(e)$ is the vector parallel to the joint's slip plane so that the more a KC and a joint are parallel we could have the smaller value. Then, finding the optimal cut-set for a_1 is equivalent to solving the well-known minimum cut problem with specified source and target node which are the two nodes the a_1 is connecting. In this manner, the found cut-set will always cut a_1 while it partitions the liaison graph into two pieces. We can prevent this cut-set from cutting other KCs by assigning a very big number to the other KCs, which will make the solution exclude the other KCs. After we iterate this procedure for all KCs existing in the subassembly, we can decide the optimal partitioning.

Although this method does not guarantee the optimality of solutions, it is moderately fast and appropriate for the problem because the KC not chosen at one partitioning always has smaller cost at the subsequent partitioning which is always conducted on a smaller graph with the same KC. Therefore, this method provides the KC with very high cost at the initial partitioning with better chance to be cut with lower cost.

3.4 Example: Subassembly partitioning for a given synthesized assembly

Suppose we have a synthesized assembly from GA, of which the liaison graph $L_0=(V_0,E_0,A_0)$ shown in Figure 5 (c). In other to partition V_0, we should formulate two minimum cut problem shown in Figure 6 as L_0 has two KCs. In each figure, the nodes with thick line represent source and target node that KC of interest used to connect. We can notice that a big number, 100 is assigned to the other KC in order to exclude it from the cut-set found. The numbers assigned to edges are the same in both figures, because the directions of KC1 and KC2 are identical. As the example is simple enough, we can easily find that {(1-2, 6), (1-2, 3-4)} with the cost of 0 is the optimal cut-set for (a) and {(3-4, 7), (6, 7)} with the cost of 0 for (b). Since the costs are equal for both problems, we can choose either KC1 or KC2. Suppose we choose KC1. Then, the partitioning results in a pair of subassemblies {1-2} and {3-4, 5, 6, 7}. The subassembly {1-2} can not be partitioned further as it consists of single part without any KC. The subassembly {3-4, 5, 6, 7}, however, still has KC2 and it has to be partitioned again. Obviously, the minimum cut is {(3-4, 7), (6, 7)} partitioning {3-4, 5, 6, 7} into {7} and {3-4, 5, 6}. Now, both subassemblies do not have any KC, the subassembly partitioning can be finished. The binary tree in Figure 7 shows the outcome of the subassembly partitioning process for the example, where the cost and the broken KC of each partitioning are annotated under the corresponding hyper-edge. The summation of the cost involved with every portioning is 0 in this partitioning, which means the synthesized assembly with this subassembly partitioning achieves the adjustability for every KC perfectly.

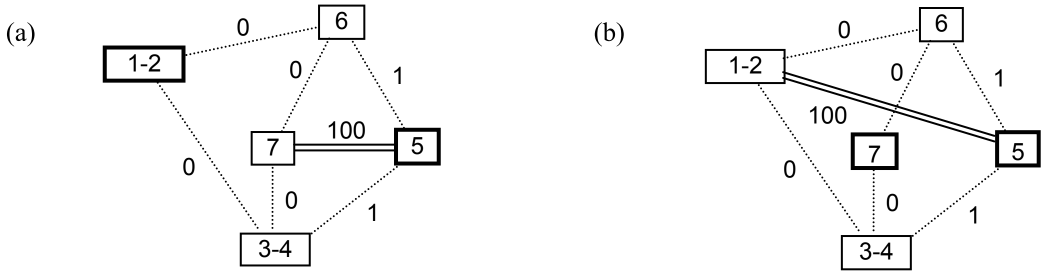

Figure 6. Minimum cut problem for the optimal cut-set for KC1 (a) and KC2 (b) for the example in Figure 5.

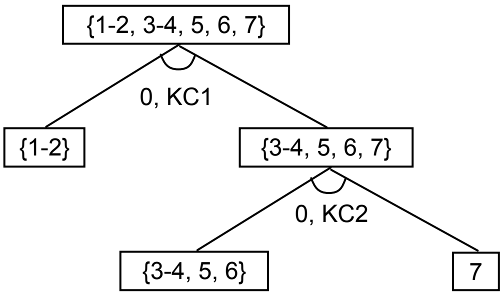

Figure 7. The subassembly partitioning decided by the dynamic approach for the example in Figure 5.

4. 3D automotive space frame example

The presented method can be applied to three-dimensional structures such as the automotive space frame model shown in Figure 8. A set of typical joint configurations available for this type of structure [8] is listed in Figure 9, where joints with a common adjustability are combined and treated as a single element in the joint library coded for the GA. Available joint configurations are dependent on the orientation of members connected by it. For example, the butt joint allowed for (nearly) perpendicular connections is not available for coaxial connections. For joints with plane adjustability such as the simple lap joints and butt joints, the normal vector of the plane has been used to compute the inner product with KC, without subtracting it from 1. Hence the cost assigned to those edges is still 0 when the KC is parallel to the plane of adjustability.

Because the model is loosely constrained in that the number of joints is much larger than that of KCs to achieve, two minor criteria have been added to the objective in order to narrow optimal solutions. The two criteria added come from manufacturability considerations and include the number of parts and the summation of the angle that every pair of connected members have in 0 to 2 scale (0 for 180 degree and 2 for 0 degree). It has been assumed that the smaller number of parts decreases the effort for handling and assembly, the larger angle of connection decreases the effort to manufacture each part (straight beams are preferred most). These additional criteria can be evaluated directly from the initial liaison graph, while the adjustability, on the other hand, is obtained through the optimal subassembly partitioning routine. The optimal solution for the model in Figure 8 is presented in Figure 10 along with its optimal subassembly partitioning shown in Figure 11.

The computer software for this problem was written in C++ with intense use of data types and algorithms of LEDA developed at Max-Planck-Institut für Informatik, Saarbrücken, Germany. The Genetic Algorithm within the software uses GAlib library developed at MIT CADLAB. For the minimum cut problem, we have adopted the C source codes written by Edward Rothberg, which implement Goldberg and Tarjan's maximum flow algorithm [9].

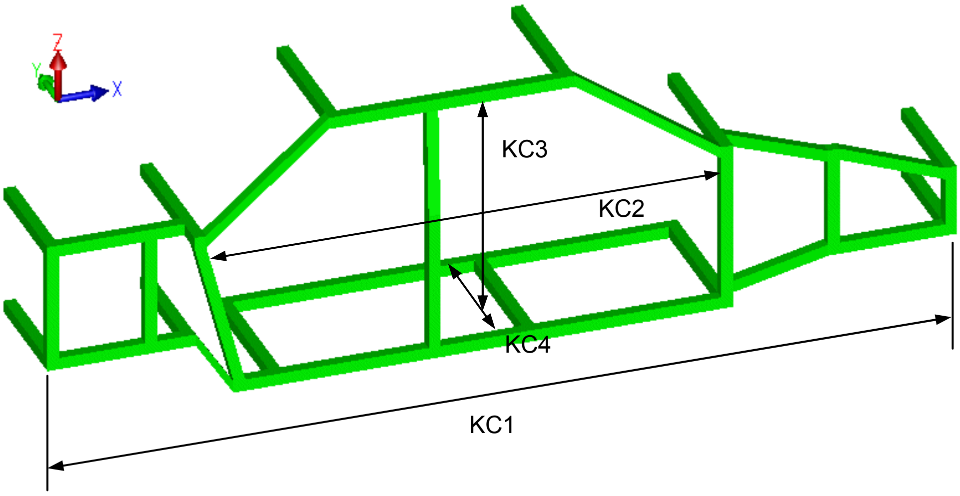

Figure 8. A three-dimensional automotive space frame model with four KCs.

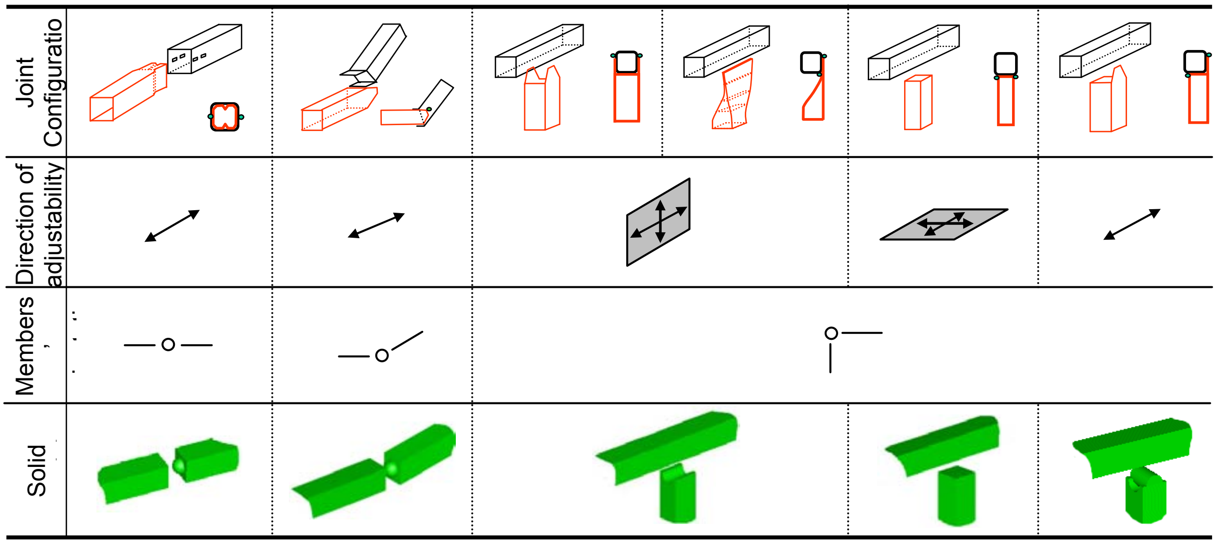

Figure 9. A joint library for automotive space frame [8].

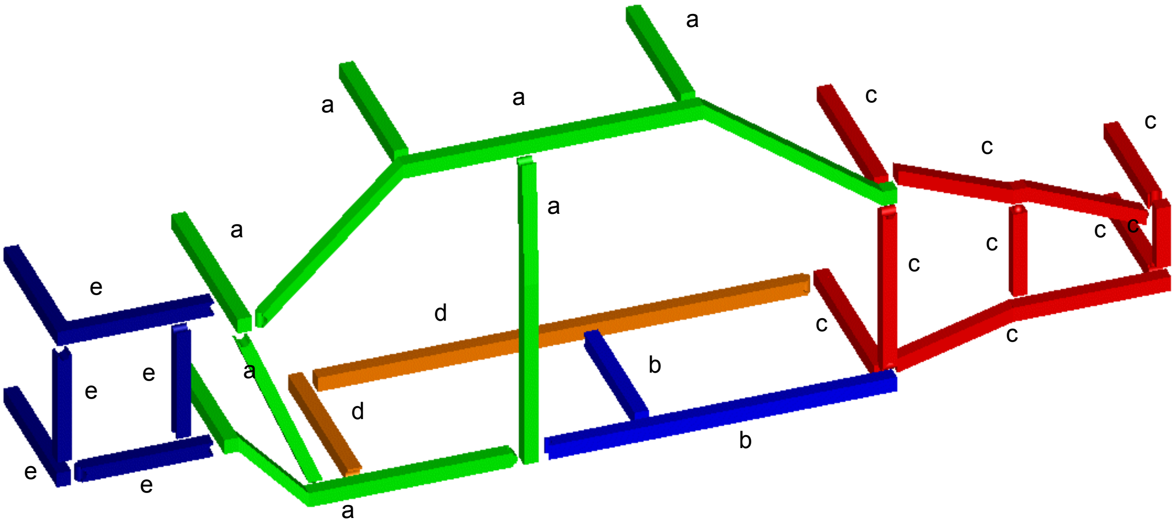

Figure 10. A synthesized assembly from the model in Figure 8 for the optimal in-process adjustability.

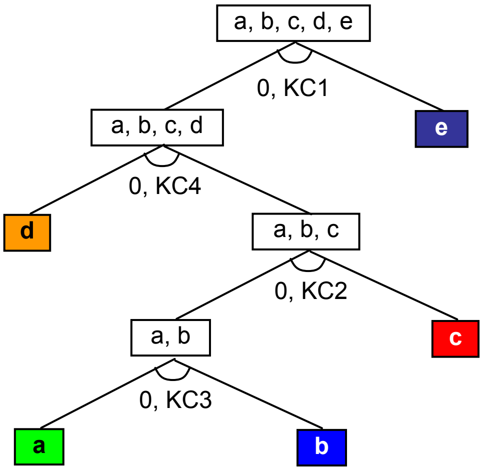

Figure11. The subassembly partitioning for the solution presented in Figure 10.

5. Summary and future works

This paper presented a GA-based method of assembly synthesis focused on the optimal in-process dimensional adjustability. In this method, the GA generates synthesized assemblies based on a specific joint library available for the application. Each synthesized assembly given by the GA undergoes subsidiary optimization routine where the subassembly partitioning is decided for the optimal in-process adjustability by recursively applying a partitioning rule. The partitioning rule at every step is transformed to the well-known minimum cut problems. Finally, the GA chooses the optimal assembly synthesis based on the adjustability returned from the subassembly partitioning. The method was applied to a three-dimensional automotive space frame model with the accompanying joint library.

The subsidiary optimization routine for the subassembly partitioning could be used alone in generating optimal assembly sequences for existing assemblies. However, if it is to be used alone, a more efficient dynamic programming is desired as the compactness of the algorithm can be sacrificed to guarantee the optimality. A three-dimensional model with a joint library could be also synthesized by the generative method using AND/OR graph of assembly synthesis [2], which also considered how properly parts are constrained during assembly operations, in addition to the adjustability.

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