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MARGINS OF PERFORMANCE IN ENGINEERING: THE REQUIREMENT FOR A SYSTEMATIC APPROACH

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Abstract

Design margins provide additional performance capability above required basic performance and may be specified by a system designer to compensate for uncertainties. It is commonplace to account for uncertainties by introducing design margins based on experience, however, times are changing and there is currently much interest in the application of systematic methods, that is, specifying a design margin based upon theory and not practical experience. The primary objective of this paper is to present a survey of existing knowledge and approaches related to the management of design margins across a broad range of engineering domains. Particular attention is given to design margins in a performance context, i.e. design margins intended to improve the probability of a system/process performing to requirements. Furthermore, based on the surveyed literature, several prevalent concepts pertaining to the management of design margins will be identified. Such prevalent concepts are significant because they form the basis of and justify the requirement for a quantitative approach. Indeed, several quantitative approaches are identified and surveyed which seek to evaluate such design margins as an alternative to their empirical selection.

Keywords: Design Margins, Uncertainty, Probabilistic Design

1 Introduction

Few engineered systems are designed with the benefit of complete information; consequently the assurance of performance can seldom be perfect. Moreover, many decisions during the design process are invariably made under conditions of uncertainty. Therefore, the accumulation of prediction errors (due to environmental uncertainty or measurement inaccuracy for example), omissions and design iterations during the design process may occur, which in turn affects the probability of the given system not performing to requirements. For example, Hockberger[3] with reference to marine engineering stated that "a designer faces a range of uncertainties (which decrease in magnitude as the design progresses) as to the ultimate physical characteristics of the ship best capable of satisfying a certain set of customer requirements" [3]. To provide some protection from these sources of uncertainty, the designer attempts to anticipate these uncertainties with a design margin. That is, he or she designs the ship with for example, enough extra space, structural strength or additional performance capability.

Design margins can be employed to increase the likelihood that a specified level of performance can be attained by a given system. That is, they are the additional performance capability incorporated into a system to compensate for uncertainties, termed within this paper as margins of performance. Frequently, design margins are empirically selected, i.e. they are specified based on experience and/or standard practice and not through theory or systematic methods. For example, Slager et al.[4] discussed how the service margin, (a propulsive margin) for United States Navy ship designs, is customarily assumed to be 25% [4]. However, within the literature documented in this paper, this practice is argued to be inappropriate due to the potential high costs associated with design margins and furthermore, such margins can be over excessive or disproportionate in nature.

A discussion follows of a number of noteworthy papers available within the literature, all of which consider margins in a performance context. Sections 2, 3 and 4 consider literature available within aeronautical/astronautical engineering, marine engineering, chemical engineering and power generation respectively. In surveying such literature, a number of key issues related to the quantification of margins of performance are identified such as the use of probabilistic design methods to rationally and analytically make design decisions in the presence of uncertainty. The results of this survey are discussed in section 5 where the prevalent concepts and key issues are summarised. Furthermore, the requirement for a more generic approach for the quantification of margins of performance in engineered systems is identified. Finally, section 6 presents current research being conducted at the University of Newcastle upon Tyne's Engineering Design Centre (EDC) in conjunction with BAE Systems – Systems Engineering and Innovation Centre (SEIC) at Loughborough. The objective of this research is to develop a unified, holistic and generic approach, one which satisfies all of the key issues highlighted in this paper.

2 Aeronautical/Astronautical Engineering

Cribbs[5] presented a Monte Carlo analysis method devised to determine the probability distribution of design speed for hypervelocity vehicle (Single Stage To Orbit or SSTO rocket) designs studied for the X-30 National Aerospace Plane program. The method presented is based upon linear perturbation theory on velocity and mass fraction models and uses uncertainties in fundamental vehicle performance variables such as drag, specific impulse, empty weight and the vehicles performance sensitivity to each variable. Furthermore, the method is used to develop vehicle performance requirements which provide the necessary margins to ensure the success of the SSTO mission.

A Monte Carlo computer program was developed by Cribbs[5] which randomly selects a perturbation value for each of the design variables considered such as combustor efficiency and zero-lift drag coefficient and simulates the given mathematical models representing velocity and mass-fraction. In aerospace engineering, the mass fraction of a rocket is an important measure of its efficiency and it measures the total amount of mass delivered to orbit as a fraction of the weight of the fully fueled vehicle prior to launch. In the case of Single Stage to Orbit vehicles, the mass fraction is simply the weight of the vehicle empty compared to full. Mass-fraction vs. velocity performance curves are then presented for each Mach regime. Furthermore, the variables are also perturbed at their nominal values or the standard design configuration. After many simulations (1000 in the example presented) a probability distribution of velocities attained is plotted (see figure 2.1). Cribbs[5] stated that the resultant probability distribution of maximum attainable velocity can be closely approximated by a normal distribution which has the familiar bell shape and its sample space extends from minus to plus infinity. Cribbs[5] stated "this is a common result when many distributions are combined" [5].

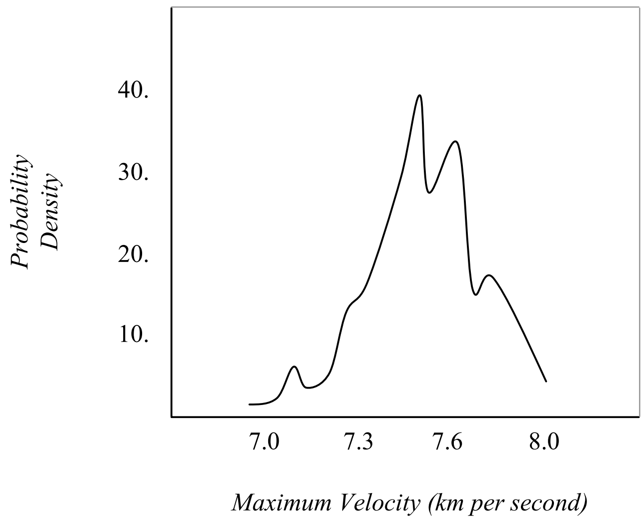

Figure 2.1. Probability Density Distribution of Maximum Velocities [5]

In making this approximation, it is assumed by the authors that Cribbs[5] was referring to the Central Limit Theorem. In an engineering context, Bury[7] discussed how the normal distribution must be approached with caution, particularly if inferences will focus on the tails of the distribution as is often the case in engineering. Furthermore, the normal distributions sample space extends from minus to plus infinity which may not be appropriate in many engineering contexts. However, Bury[7] stated that “on the basis of a central limit theorem the normal distribution is the model of choice if it can be argued that the random variable under consideration is the aggregate sum of many [independent] underlying causes” [7]. With regards to postulating the central limit theorem, Crawshaw and Chambers[8] suggested that the sample size of independent underlying causes should be greater than 30.

Using the probability distribution of maximum attainable velocity, the probability that the SSTO vehicle can achieve a particular maximum velocity can be estimated. In the example presented by Cribbs[5] the probability of reaching the design speed target (7802.88 m/s) is very low and hence the vehicle should be designed to carry more fuel than the nominal vehicle sizing. For example, a velocity margin of approximately 518.6 m/s is required for a 90% confidence of performing to requirements. This is then translated to a weight margin of 30%. However, it is not clearly expressed how the stated design margin should be allocated. That is, upon specifying how much design margin is required to achieve a specified level of performance, information regarding how it should be allocated into the SSTO vehicle is not well defined. The method presented by Cribbs[5] is a specialised method that is tailored for a specific problem, that is; how much velocity margin is required for an SSTO vehicle in order to achieve a given mission confidence of performing to requirements. Hence, specifying a velocity margin of 518.16 m/s or a weight margin of 30% may be appropriate for this particular problem. Finally, Cribbs[5] discussed the concept of a robust design which has built in margins to assure that a given flight vehicle will achieve its design goal.

With reference to SSTO vehicles, it is argued that [10] Computational Fluid Dynamics (CFD) is an essential part of the design process for aerospace planes and furthermore, SSTO aerospace planes with air-breathing supersonic and hypersonic combustion are going to be largely designed by means of CFD. Mehta[10] discussed how the existing data base for designing SSTO aerospace planes with supersonic and hypersonic combustion is very limited and the existing ground-based test facilities and test facilities are deficient for developing an adequate data base. "computational fluid dynamics on the other hand can go a long way toward determining the performance and specifications of these planes" [10]. Mehta[10] argued that the challenge posed by all aerospace plane design efforts is to obtain credible CFD results, to assess the probable (quantified) uncertainties in those results and to certify the codes as tools. He identified the primary requirements for credible computations as; (1)the fluid dynamics of the boundary layer, mixing and combustion are adequately modelled and (2) the necessary computing resources are available. The inability to fulfil these requirements leads to uncertainties in computed results which in turn leads to risks. It is noted [10] that addressing risk is addressing uncertainties and the credibility level of the design is, in part determined by quantifying CFD uncertainties. With this in mind Mehta[10] identified two types of risk associated with aerospace planes; (1) success risk and (2) safety risk. Success risk is defined as the probability of not achieving the objectives of the program whereas safety risk is the probability of potential failures and hazards. In the assessment and reduction of success risk, the potential design margins are considered. Furthermore the relationship between design margins and the quantification of uncertainty is recognised. Mehta[10] stated that "a margin to be built into a design requires quantification of uncertainties because the margin is a quantitative entity" [10].

Mavris et al.[9] examined ways in which to implement probabilistic design methods in the aircraft engine preliminary design process. The focus of this is to analytically determine the impact of uncertainty in engine component performance on the overall performance of a notional large commercial transport, particularly the impact on design range, fuel burn and engine weight. The emphasis of the paper was twofold:

1. To find ways to reduce the impact of uncertainty in engine component performance through appropriate engine cycle selections.
2. To find ways to leverage existing design margin in order to squeeze more performance out of current technology.

The approach presented by Mavris et al.[9] utilises standard Response Surface Methodology (RSM) in conjunction with the Fast Probability Integration (FPI) method. FPI is described as an advanced probabilistic analysis method that was developed in the early 90's at the Southwest Research Institute (SwRI) under contracts from NASA Lewis Research Centre. It is reported [9] that "FPI works by using the actual analysis code and approximates a Monte Carlo analysis as opposed to the RSM/Monte Carlo method which approximates the analysis code and then uses Monte Carlo analysis"[9]. The objective of using RSM is to create an approximate analytical model of a given data set (generated by running the existing analysis code) using Response Surface Equations (RSE) to model data behavior.

The RSEs are used to plot contours which depict the design space in a graphical and intuitive way, showing the constraints as well as where the best design range regions are located. In addition to design range, fuel burn and engine weight are also considered although design range receives the preponderance of interest. Mavris et al.[9] stated "in effect, these contours are a slice of data at a given probability level (or *p*-level) with the RSEs representing design performance at that *p*-level are constructed" [9]. An example of such a contour plot can be seen in figure 2.2.

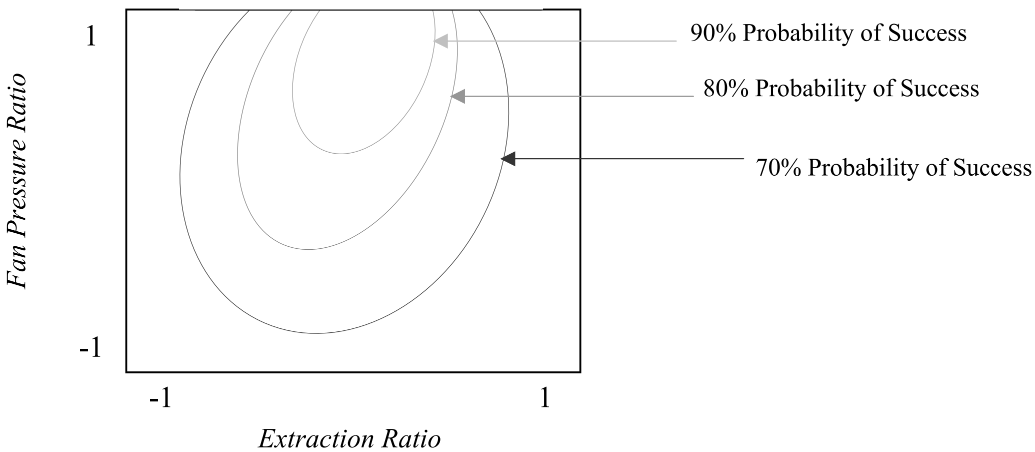

Figure 2.2. "Slicing" of CDF's to Create Probability Contours [9]

Following the steps discussed, it is argued [9] "from this point, the problem becomes an exercise of trying to find the best balance between weight, fuel burn (specific fuel consumption) and the probability of meeting the design range target while simultaneously avoiding violation of any constraints (such as limits on fan diameter)" [9]. Mavris et al.[9] noted that such a method can be used to leverage the existing design margin available in order to achieve better performance with the same technology level [9]. Finally, it can be argued [9] that probabilistic design methods provide an analytical framework for answering such questions as "how much design margin is really necessary?". An implication being the empirical selection of design margins is inappropriate.

Colbourne et al.[11] demonstrated using optimisation and trade-off studies that design margins can be systematically adjusted to quickly retune the design of a helicopter and to generate trade-off curves. With regards to the RASCAL Black Hawk helicopter it was demonstrated that a 5% design margin applied to control systems gives the best handling qualities performance without excessive actuator activity. The optimisation and trade-off studies are performed by [11] using software named "CONDUIT". CONDUIT tunes the design variables to optimise the system to the objective handling quality performance criteria whilst satisfying the Level 1 region constraints. The Level 1 region refers to aircraft handling characteristics that are satisfactory without requiring improvement and this is the desirable performance region. The CONDUIT software incorporates the RASCAL control laws and flight control system which operates the RASCAL research actuators which in turn drive the Black Hawks primary servos. The nature of this directly effects the handling qualities of the helicopter in question. Colbourne et al.[11] discussed that the Level 1 handling qualities were achieved by increasing the pitch and yaw command model bandwidths and when this was achieved CONDUIT minimised actuator energy and crossover frequency while maintaining Level 1 handling qualities.

A trade-off study was performed for the RASCAL design by looking at the effect on performance by varying the design margin. CONDUIT accommodates uncertainty in the mathematical model and changes in actual flight conditions by allowing the user to include a design margin on the helicopter flight control systems. However, although it discussed how uncertainty is accommodated in the design margin trade-off, no detail is provided to the nature of the analysis, that is; the type of uncertainty considered, how uncertainty is quantified or any estimation to the probability of the system underperforming. The design margin in this case enforces overdesign to ensure that acceptable solutions lie a set distance into the Level 1 region and not on the Level 1/Level 2 boundary (see figure 2.3). It is stated that [10] this builds in “design robustness”. However, the design margin is not considered by the author to minimise performance variance, only to provide a region to which performance can deviate into without crossing the Level 1/ Level 2 boundary.

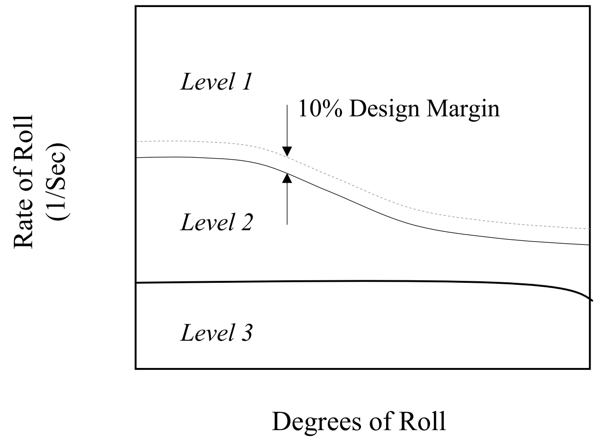

Figure 2.3. Handling Quality (quickness of roll) and design margin [10]

3 Marine Engineering

With regards to marine engineering, two major categories of design margin are identified by [12]; Design and Construction Margins (D&C Margins) and future growth margins. D&C margins are allowances made during the preliminary design of a ship due to unknowns associated with the design process. Future growth margins are primarily allowances made at the request of a customer in anticipation of the future installation in a ship of items that are not required at the time of construction. Furthermore, they enable the addition of equipment during modernisation or conversion thus enhancing flexibility and adaptability. A third major classification is identified by [3]; the assurance margin. Assurance margins are defined by [3] as a “key element in the probability of a system being able to perform to requirements; that is, to attain a specified level of performance under specified conditions” [3]. In agreement, Garzke and Kerr[13] stated how assurance margins are employed to ensure that a specified level of performance can be attained during the operational life of a ship. Therefore, assurance margins are primarily concerned with operational capability and performance rather than predominantly with design, as is the case with future growth and D&C margins.

Hockberger[3] illustrated an assurance margin for the general case where the load for which a system is being designed is not a single known value, but rather a range of possible values forming a distribution having some mean μ and some standard deviation σ. The capability of the system to carry that probabilistic load is again not a single known value, but a range of possible values forming a second distribution. It can be seen in figure 3.1, from the shaded overlapping region, that it is possible to have loads that exceed the systems capability resulting in underperformance.

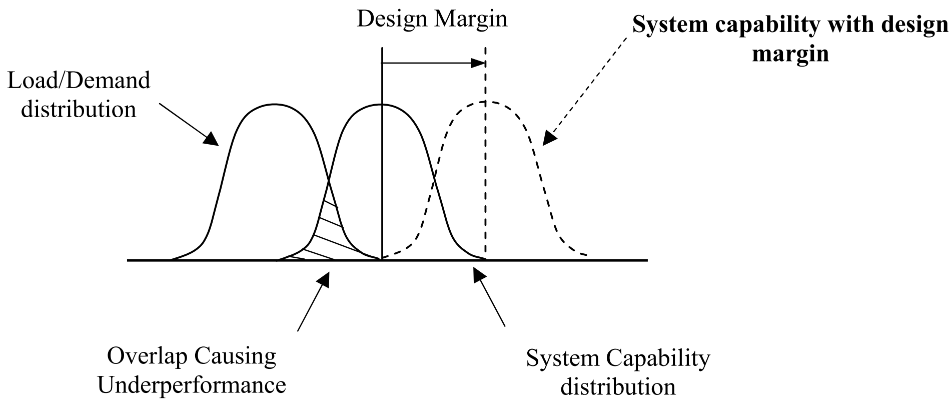

Figure 3.1. Ilurstration of the asurance margin

Hockberger[3] stated how the probability that the system will meet its requirements can be increased by causing the two distributions to move further apart as illustrated in figure 3.1. The assurance margin therefore can be defined as the measure of separation of the two distributions of load/demand and capability. Frequently, such design margins are empirically selected and it is discussed [3] how a standard 25% assurance margin on power is common practice in ship propulsion design. In agreement, Levine and Hawkins[15] discussed how such margins are empirically selected and in addition, argued that “the empirical selection of design margins is inappropriate and represents a large investment, one which warrants thorough justification” [15]. Further evidence of considerable power margins in ships can be found within the literature when considering hydrofoil craft. U.S. Navy hydrofoil craft are specified with a range of assurance margins on power of the order 20% - 50% [14]. However, because the magnitude of this margin is a prime factor in the sizing of the propulsion system, it is essential that it is not arbitrarily overspecified [14].

Finally, Hockberger[3] argued that the application of a standard assurance margin ignores important differences in the physical characteristics of individual systems and the purposes for which those systems are designed. Hence a standard margin such as 25% may not be effective in all cases. An example was provided: Even between two ships of the same size and sea response, the value of a particular probability of making the required speed may differ considerably depending on their respective missions.

4 Chemical Engineering and Power Generation

With reference to uncertainty in chemical and environmental systems, [19] discussed how uncertainty always exists in such systems and how there is a problem in evaluating their effects. It is discussed [20] how these and other related problems are overcome by considering appropriate design margins. It is highlighted that design margins are provided in a chemical plant for the following reasons:

- To take care of any variations in input/output material specifications,
- To take care of any operational problems,
- To take care of any unknown factors influencing process plant capacity or product quality,

- To take care of any fabrication/equipment selection errors.

Furthermore, [20] suggests a cautious approach when selecting appropriate design margins so that all the benefits are taken up while disadvantages are minimised.

Dittmar and Hartmann[16] stated that, "in the design of process units and systems, design margins are often added to the design variables in order to obtain a preventative compensation for parameter uncertainties in the mathematical models used for the design". Furthermore, it is discussed how design margins exert a decisive influence on the cost of chemical plants and with chemical plants becoming more complex, the empirical estimation of design margins becomes unjustified [16]. Dittmar and Hartmann[16] presented a method for the optimal estimation of design margins in the sense of minimisation of systems reserves based on a proposition by [17] and [18]. Takamatsu et al[17] stated "it is usual in practical design of engineering systems to add some margin estimated from experience to the design variables obtained from theoretical or mathematical equations". However, they discussed how the value of design margins can be quantitatively estimated by using the concept of sensitivity and linear programming.

Dittmar and Hartmann[16] demonstrated the application of the method to a reactor-separator system. The system consists of a Continuous Stirred Tank Reactor (CSTR) of volume V and a separator. In this example the desired product produced by the system is a raw material termed R. Dittmar and Hartmann[16] developed a mathematical model of the reactor-separator system and a set of associated boundary conditions of the model. One boundary condition to be satisfied in this case is that the raw material R should be produced at a rate of 70mol/h. The determination of the design margin in this case involves optimising the mathematical model (by means of LP) where the intention is to satisfy the boundary conditions of the system (material R to be produced at a rate of 70mol/h) which may be compromised due to the effect of a number of uncertain variables (reactor rate constants K_B and K_X). This is achieved through appropriately increasing the reactor volume V and simultaneously minimising the associated increase in system cost. The increase in reactor volume constitutes the design margin and the most undesirable system state occurs when the uncertain variables (reactor rate constants K_B and K_X) are located at their respective lower limits.

5 Conclusion

The issue of design margins, specifically margins of performance, has received the attention of many researchers from a variety of engineering domains including marine engineering, aeronautical/ astronautical engineering and chemical engineering. With regards to margins of performance, a number of prevalent concepts have been identified across the aforementioned domains as follows:

- Uncertainty is identified as a primary driver for the requirement of design margins. Furthermore, with reference to aerospace design, [10] stated that "in order to make rational decisions regarding the placement of design margins, uncertainty should first be evaluated in a quantitative manner".
- The empirical selection of design margins is highlighted as being inappropriate due to the potential high costs associated with them. For example, [3]] discussed the standard practice of using a 25% Assurance Margin on propulsion power in the design of ships. Furthermore, [16] discussed how design margins exert a decisive influence on the cost of chemical plants and with chemical plants becoming more complex, the empirical estimation of design margins becomes unjustified.

Such prevalent concepts are significant because they form the basis of and justify the requirement for a quantitative approach for the evaluation of margins of performance in engineered systems. Indeed, several quantitative approaches have been identified and surveyed which seek to evaluate such design margins as an alternative to their empirical selection. In addition to identifying a number of prevalent concepts, a number of key issues related to the surveyed approaches are identified as follows:

1. Quantification of uncertainty has been identified with respect to the probabilistic approach in the evaluation of design margins [5, 9; 21, 3],
2. Mathematical modelling and simulation underpin the analysis of systems when considering uncertainty, probabilistic design and the quantification of design margins [5, 16, 3, 15, 21, 9],
3. Probabilistic design methods are utilised to rationally and analytically make design decisions in the presence of uncertainty and with regards to the quantification of design margins [9],
4. It is argued that there is scope to incorporate robust design when quantifying and allocating design margins in the sense of minimising performance variance [5, 11],
5. Finally, optimisation can be utilised to accomplish the following:
    - The minimisation of performance variance [11].
    - The leverage of performance whilst simultaneously satisfying the design margin requirement, that is; a design margin can be allocated in an optimal manner [16].

In summary, the specific literature surveyed in this paper provide valuable contributions in the field of design margins and more specifically, margins of performance. However, the actual approaches for the quantification of performance margins surveyed in this paper are limited in their application. That is, they are designed to address a specific problem within their given engineering domains. Hence, on this basis, the main outcome of this literature survey is that there is scope for a more generic and systematic approach which considers collectively the aforementioned key issues. What follows is a brief but concise overview of the current approach. However, much discussion regarding the mathematical and statistical background is excluded due to space constraints.

6 An Approach for the Quantification of Margins of Performance

The approach described here enables metamodels (models of models) to be developed of given systems from existing computerised models through the utilisation of response surface based robust design techniques. In general, using design of experiments, a given computerised model can be simulated or exercised for given settings of the model variables. From the generated data, polynomial regression models can be fitted accordingly using standard Response Surface Methodology (RSM). RSM comprises a group of statistical techniques for empirical model building. By careful design and analysis of experiments or model runs, it seeks to obtain a simplified relationship between a number of predictors, or input variables and an output or response variable. Intuitively, such a response of interest is considered as the performance characteristic under investigation. In accordance with the response surface approach to robust design, model variables are classified as either noise (z_i) or control variables (x_i). Control variables refer to those whose settings can be changed by the designer whereas noise variables are those that display inherent variability and/or the designer has no direct control.

With this in mind, the approach seeks to generate a response model of the form

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{r} \delta_i z_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{i=1}^{k-1} \sum_{j>i}^{k} \beta_{ij} x_i x_j + \sum_{i=1}^{k} \sum_{j=1}^{r} \gamma_{ij} x_i z_j + \varepsilon \tag{1.1}$$

where y is the response of interest, β_i, β_{ii} and β_{ij} are the regression coefficients for the control variables x_i, their quadratic terms and interactions respectively, δ_i are the regression coefficients for the noise variables z_i, γ_{ij} are the regression coefficients for the interactions between the noise and control variables and finally, ε is the error due to regression. Using the transmission of error approach as discussed by Montgomery[21] (expanding equation 1.1 in a first order Taylor series expansion about $z_i = 0$), models for expectance and variance can be derived as follows

$$E_z[y(x,z)] = f(x) \tag{1.2}$$

$$V_z[y(x,z)] = \sigma_z^2 \sum_{i=1}^{r} \left[\frac{\partial y(x,z)}{\partial z_i} \right]^2 + \sigma^2 \tag{1.3}$$

Where $f(x)$ in (1.2) is the portion of the model that involves only the control variables (with respect to equation 1.1), in equation (1.3) σ_z^2 is the variance and σ^2 is the error due to fitting the original regression model, i.e. equation (1.1). Due to the first - order Taylor series expansion, there are no noise variables present in equations (1.2) and (1.3). Hence, the engineer can potentially set the control variables to achieve a target value of the expectance and minimise the variability transmitted by the noise variable. Furthermore, although the variance model involves only the control variables, it does contain the interaction regression coefficients (δ_i and γ_{ii}) between the control and noise variables. This is how the uncertainty transmitted by the noise variable influences the response y. The principal advantage of utilising RSM is that the fitted polynomial regression model can be used as a replacement or proxy for the computerised model (valid for the design space regressed) and all inferences related to uncertainty analysis and optimisation are derived from this fitted model. Furthermore, design of experiments and regression analysis have been applied to a broad range of engineering disciplines, such as the design of mechanical components [22], reinforced concrete columns [23] and examining aircraft concept feasibility and viability [24]. Provided RSM can be applied to a given system, the approach described here is potentially applicable to a variety of systems and is hence generic in nature.

Considering the prevalent concepts and key issues identified within this paper, it was established that uncertainty should be evaluated in a quantitative manner. Hence, the approach includes appropriate uncertainty analysis procedures. Upon deriving models for the expectance and variance of the response of interest, a description of the uncertainty in the performance capability of the given system under investigation is required. More specifically, the probability the given system will underperform (with respect to a specified performance requirement) is desired. In order to do so, the approach exploits Chebyshev's inequality and the Vysochanskii - Petunin inequality for estimating the probability of underperformance of a given system. Chebyshev's Inequality is a result in probability theory that places an upper bound on the probability that a random variable will differ from its mean by more than a fixed number t.

If we let a random variable X have expectance $E(\mathrm{X})$ and variance $Var(\mathrm{X})$ and let t be any positive number

$$P\left[\left|X - E(X)\right| \geq t\right] \leq \frac{Var(X)}{t^2} \tag{1.4}$$

With regards to the approach, it is possible to obtain an estimate of the probability of underperformance provided we define the performance characteristic under investigation (modelled using a second - order response model) y as a random variable with expectance $E_z[y(x, z)]$ and variance $V_z[y(x, z)]$. By doing so, Chebyshev's inequality can be employed to place an upper bound on the probability that a given system will generate an undesirable level of performance. For example, consider the second - order response model (1.1) and the corresponding models for its expectance and variance (1.2) and (1.3). If we define a performance requirement placed on the system under investigation as R_y ($R_y > 0$) the probability of underperformance is given by (1.5). That is, an upper bound can be placed on the probability that a given system will underperform with respect to the pre - defined performance requirement R_y. Note that R_y is corrected for the expectance $E_z[y(x, z)]$, that is, we are calculating the probability that the performance will differ from the mean a distance of $| R\mathrm{y} - E_z[y(x, z)]|$, that is, $t \geq \{| R_y - E_z[y(x, z)]|\}$. Note, $| R_y - E_z[y(x, z)]|$ gives the modulus of $R_y - E_z[y(x, z)$, i.e. the magnitude irrespective of sign.

$$P\left[\left|y - E_z\left[y(x,z)\right]\right| \geq t\right] \leq \frac{V_z\left[y(x,z)\right]}{t^2} \tag{1.5}$$

Chebyshev's inequality has the prime advantage of universality, that is, it applies to all random variables whose first and second moments exist [26]. Equivalently, the theorem applies to non - bell shaped distributions and therefore, no assumption on the output distribution in an uncertainty analysis is required [27]. Some criticism of Chebyshev's inequality exists within the literature, specifically, the theorem can provide loose bounds on the probability that a random variable will differ from its mean by more than a fixed number t. However, tighter bounds (on the probability of underperformance) can be generated on that produced by Chebyshev's inequality using the Vysochanskii - Petunin inequality. It is noted [27] that if the unimodal probability distribution or density function is not symmetric then the inequality applies only for high t, that is, when $t^2 > B^2. Var(\mathrm{X})$ where B is approximately equal to 1.38539[28]. If this is not the case however, Chebyshev's inequality is appropriate. The probability of underperformance using the Vysochanskii - Petunin inequality is given by (1.6), where the distribution is assumed unimodal and $t^2 > B^2. Var(\mathrm{X})$.

$$P\left[\left|y - E_z\left[y(x,z)\right]\right| \geq t\right] \leq \frac{4.V_z\left[y(x,z)\right]}{9.t^2} \tag{1.6}$$

Upon estimating the probability of underperformance of a given system, the approach then seeks to calculate how much additional performance capability is required to improve the probability of performing to requirements. Suppose an engineer desires a given system to perform to requirements with a probability of P. Equation (1.6) can be re - defined in order to estimate what expected performance ($E_z[y(x, z)]$) would indeed ensure that a given system would perform to requirements with a probability of less than P_R. That is, the engineer wishes to know by how much does the expected performance of a given system need to be improved? For the purposes of the approach, this expected performance is defined as the target expected performance $T_{Ez}[y(x, z)]|$.

This can be found by solving (1.7) for the target expected performance $T_{Ez}[y(x, z)]$ when P_R, $V_z[y(x, z)]$ and R_y are known.

$$P_R \le \frac{V_z[y(x,z)]}{\{R_y - T_{E_z[y(x,z)]}\}^2} \tag{1.7}$$

To solve (1.7) we express the given inequality with zero on the right hand side and then determine the roots defined by $T_{Ez}[y(x, z)]$. For (1.7) this gives

$$T^2_{E_z[y(x,z)]} - 2R_y T_{E_z[y(x,z)]} + R_y^2 - \left(\frac{V_z[y(x,z)]}{P_R}\right) \le 0 \tag{1.8}$$

For example, Suppose the gross thrust of a jet engine y (modelled using a second - order response model) has expectance $E_z[y(x, z)]$ 16325 N and variance $V_z[y(x, z)]$ 235936 N^2. Furthermore, a performance requirement R_y is specified to be ≥15124 N. The probability of the engine producing 15124 Newtons or less (i.e. underperforming) is given by Chebyshev's inequality as follows

$$P\left[|15124 - 16325| \ge t\right] \le \frac{235936}{(15124 - 16325)^2} \quad \textit{that is} \quad P\left[|15124 - 16325| \ge t\right] \le 0.16 \tag{1.9}$$

That is, probability of underperformance is estimated to be no greater than 16%. The quoted Chebyshev inequality is known as the two tailed version and thus the calculated probability of failure is worse case as it assumes that the thrust distribution is skewed to the pessimistic extreme. Now suppose an engineer desires a 95% probability that the engine will perform to requirements, i.e $P_R = 0.05$. The target expected thrust $T_{Ez}[y(x, z)]|$ that will achieve this aim is estimated using (1.10) and (1.11). The calculation is as follows

$$0.05 \le \frac{235936}{\{15124 - T_{E_z[y(x,z)]}\}^2} \tag{1.10}$$

$$\textit{hence} \quad 228735376 - 30248 T_{E_z[y(x,z)]} + T^2_{E_z[y(x,z)]} \le 0 \tag{1.11}$$

The roots of (1.11) are $T_{Ez}[y(x, z)]| = 17296$ or $T_{Ez}[y(x, z)]| = 12952$. Therefore, intuitively, the desired target expectance $T_{Ez}[y(x, z)]|$ is 17296 N of gross thrust or an additional performance capability of $T_{Ez}[y(x, z)]| - E_z[y(x, z)] = 971$ N. More specifically, a performance margin of 971N (a 5.9% increase) gross thrust is required. Confirming that if the jet engine can be modified to produce an expected gross thrust of 17296 N, the probability of underperformance is no greater than 5%, i.e. a 95% probability of performance to requirements. Finally, current research at the Engineering Design Centre is now focussing upon a method to allocate a performance margin once its requirement has been established. It is proposed that this can be achieved using an appropriate optimisation strategy where the optimisation problem can be posed as follows:

Minimise $V_z[y(x, z)]$

Subject to

$E_z[y(x, z)] = T_{Ez}[y(x, z)]|$

$a \le x_i \le b$

That is, to simultaneously minimise performance variance whilst achieving the target expected performance through the manipulation of the control variables x_i.

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