INTERNATIONAL DESIGN CONFERENCE - DESIGN 2004
Dubrovnik, May 18 - 21, 2004.

MULTIOBJECTIVE OPTIMIZATION IN INDUSTRIAL DESIGN

F. Cappello and M. Marchetto

Keywords: multiobjective optimization, genetic algorithms

1. Introduction

Many real-world problems involve simultaneous optimization of several incommensurable and often competing objectives. In these cases, generally, there is not a single optimal design, but rather a set of alternative solutions. These solutions, known as Pareto-optimal points [Pareto, 1896], are optimal in the wider sense that no other solution in the search space is superior to them when all objectives are considered. Since the mid-1980s, there has been a growing interest in solving multiobjective problems using genetic algorithms because they process a set of solutions in parallel allowing to obtain the Pareto Set through a unique run. In this study, we propose a new genetic approach to multiobjective optimization, named SPLSDCAS (*Strength Pareto, Local Selection, Directional Crossover, Additive Sharing*) in order to fill the gaps of the current best genetic techniques. Besides, we provide a comparison of SPLSDCAS and Poloni-Pediroda technique [Quagliarella et al., 1997] whereas a weighted sum approach serves as additional point of reference. We show that SPLSDCAS outperforms the other algorithms under consideration in identifying the Pareto optimal frontier.

2. Multiobjective optimization using genetic algorithms

2.1 Definition of the multiobjective optimization problem

Formally, the multiobjective optimization problem can be completely described by a vector function $\boldsymbol{f}$ which maps a vector of m decision variables to a space of n objectives:

$$\begin{aligned} &\textit{minimize/maximize}\ \ \boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x}) = (f_1(\boldsymbol{x}), f_2(\boldsymbol{x}),..., f_{\boldsymbol{n}}(\boldsymbol{x})) \\ &\textit{subject to}:\ \boldsymbol{x} = (x_1, x_2,..., x_{\boldsymbol{m}}) \in X,\ \boldsymbol{y} = (y_1, y_2,..., y_{\boldsymbol{n}}) \in Y \end{aligned} \tag{1}$$

where $\boldsymbol{x}$ is called the *decision vector*, X is the *parameter space*, $\boldsymbol{y}$ is the *objective vector*, and Y is the *objective space*. Conceptually, the set of optimal solutions of a multiobjective problem consists of all decision vectors for which the corresponding objective vectors cannot be improved in any dimension without degrading in another: these decision vectors are named *Pareto optimal solutions*.

Formally, assume, without loss of generality, a minimization problem and consider two feasible solutions $\boldsymbol{x}^{(1)}, \boldsymbol{x}^{(2)} \in X$: $\boldsymbol{x}^{(1)}$ is said to dominate $\boldsymbol{x}^{(2)}$ (also written $\boldsymbol{x}^{(1)} \prec \boldsymbol{x}^{(2)}$) if

$$\forall i \in \{1,2,...,n\}: f_{\boldsymbol{i}}(\boldsymbol{x}^{(1)}) \le f_{\boldsymbol{i}}(\boldsymbol{x}^{(2)}) \wedge \exists\, j \in \{1,2,...,n\}: f_{\boldsymbol{j}}(\boldsymbol{x}^{(1)}) < f_{\boldsymbol{j}}(\boldsymbol{x}^{(2)}) \tag{2}$$

All solutions, that are non-dominated within the entire search space, are considered *Pareto optimal* and constitute the *Pareto Set* in the parameter space and the *Pareto Frontier* in the objective space.

2.2 The preservation of diversity

Due to stochastic errors associated with its operators, genetic algorithm tends to converge to a single solution when it is used with a finite population. In a multiobjective optimization problem, we want to find the entire Pareto Set and not only a single non-dominated solution, so this convergence phenomenon, named *genetic drift*, has to be avoided by means of a technique of preservation of the diversity in the population. In this study, we consider the two most popular approaches used in order to maintain the population heterogeneity: *Local Pareto Selection* and *Fitness Sharing*.

2.2.1 Local Pareto Selection

Poloni and Pediroda [Quagliarella et al., 1997] proposed an interesting technique able to maintain diversity, named *Local Pareto Selection*, which basically consists of placing the population on a toroidal grid and choosing the members of the local tournament by means of a random walk in the neighbourhoods of a given grid point.

2.2.2 Fitness Sharing

Goldberg and Richardson [Goldberg and Richardson, 1987] suggested a different approach, named *Fitness Sharing*. This technique is based on the idea that individuals in a particular niche have to share the available resources, so the more individuals are located in the neighbourhoods of a certain design, the more its fitness value has to be degraded. They defined the following *sharing function*:

$$\phi(d_{ij}) = \begin{cases} 1-(d_{ij}/\sigma_{sh})^{\alpha} & , \ if\ d_{ij} < \sigma_{sh} \\ 0 & , \ otherwise \end{cases} \tag{3}$$

where normally $\alpha = 1$, d_{ij} is a measure indicative of the distance between solutions i and j, and σ_{sh} is the parameter, named *niche radius*, which controls the extent of sharing allowed. The *shared fitness* of a design i is calculated as $shared\ fitness_i = fitness_i / m_i$ where $m_i = \sum_{j=1}^{N} \phi(d_{ij})$ is named *niche count* of the solution i and N is the size of population.

3. A new approach to multiobjective optimization: SPLSDCAS

We propose a new genetic approach to multiobjective optimization, named SPLSDCAS, whose most important procedures are described in detail in the following sections.

3.1 Constraints treatment

The general multiconstraint problem is transformed into an unconstrained one using penalty functions [Srinivas and Deb, 1995]. This method is based on the concept that the fitness function has to be decreased according to the intensity of constraint violation. Assume, without loss of generality, a minimization problem with q constraints as follows:

$$\begin{aligned} &minimize\ \boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x}) = (f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), ..., f_n(\boldsymbol{x})) \\ &subject\ to\ \ g_j(\boldsymbol{x}) \le 0\ \ with\ tolerance = toler_j \quad j = 1,2,...,q \end{aligned} \tag{4}$$

The j-th constraint is transformed into a fuzzy function c_j as follows:

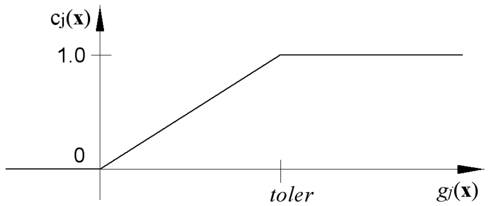

Figure 1. Fuzzy constraint treatment

Each objective $f_i(\boldsymbol{x})$ of the solution $\boldsymbol{x}$ is then penalized as follows:

$$penalized\_f_i(\boldsymbol{x}) = f_i(\boldsymbol{x}) + \sum_{j=1}^{q} c_j(\boldsymbol{x}) \cdot \max(f_i) \quad i = 1,2,\ldots,n \tag{5}$$

where $\max(f_i)$ is the maximum value that the i-th objective acquires within the entire population.

3.2 Fitness assignment

The fitness assignment procedure is based on the *Strength Pareto Approach*, but it tries to fill the gaps of SPEA procedure [Zitzler and Thiele, 1999]. First of all, we distinguish:

- Non-dominated off-line solutions, which are not dominated by any design that has been calculated till now; these solutions belong to the external Pareto Set P.
- Dominated off-line solutions, which are dominated by at least one member of the Pareto Set and which are not dominated by any individual that belongs to the current population.
- Dominated on-line solutions, inferior to at least one member of the current population.

To each solution x is assigned a real value $s(\boldsymbol{x})$, named *Pareto strength*, calculated as $s(\boldsymbol{x})=d/N$, where N is the size of population and d indicates the number of individuals in the current population which are dominated by $\boldsymbol{x}$. The anti-fitness (note that small anti-fitness values correspond to high reproduction probabilities) of a non-dominated off-line solution is equal to its strength:

$$fit(\boldsymbol{x}_p) = s(\boldsymbol{x}_p) \quad \forall \boldsymbol{x}_p \in P \tag{6}$$

The anti-fitness of a dominated (off-line or on-line) solution is calculated adding one to the strengths of all individuals j, belonging to both population gen and external Pareto Set P, which dominate it:

$$fit(\boldsymbol{x}_d) = 1 + \sum_{\substack{\boldsymbol{j} \prec \boldsymbol{x}_d \\ \boldsymbol{j} \in gen \cup P}} s(\boldsymbol{j}) \quad \forall \boldsymbol{x}_d \in gen / \exists\, \boldsymbol{j} \in gen \cup P, \boldsymbol{j} \prec \boldsymbol{x}_d \tag{7}$$

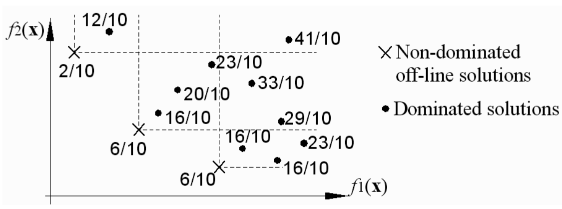

Figure 2. Anti-fitness values for a minimization problem with two objectives

This procedure, as shown in figure 2 which refers to a bi-objective minimization problem with 10 dominated individuals and 3 non-dominated off-line solutions, is able to:

- favour the individuals nearest to the Pareto optimal frontier.
- preserve diversity in the population because the individuals, having many neighbours in their niche, are penalized because of the high strength value of the associated dominant designs.
- discriminate individuals which are dominated by the same Pareto Set members, according to the dominance relation between them.
- differentiate non-dominated solutions, favouring those designs which sample unexplored regions of the feasible space.

3.3 Additive Sharing

In order to distribute uniformly individuals along the trade-off surface, we are foremost interested in maintaining diversity in the objective space. On the other hand, we also want to find diverse solutions able to achieve the same set of attribute values; therefore, the target of our sharing procedure is simultaneously to preserve diversity in the objective space and in the parameter space. First of all, we calculate the niche count m_{obj} in the attribute space, using the Holder metric of degree p=1; afterwards we calculate the niche count m_{par} in the parameter space. Finally we add the two different niche counts using a weight w_{sh}>0.5 for m_{obj} in order to emphasize the primary importance of preserving diversity in the objective space. Formally, the global niche count is calculated as $m = w_{\text{sh}} \cdot m_{\text{obj}} + (1 - w_{\text{sh}}) \cdot m_{\text{par}}$.

3.4 Selection procedure

In a first phase, the population is placed on a toroidal grid and the individuals taking part to the local tournament are chosen by means of a random walk of length R in the neighbourhoods of a given grid point. This grid point is randomly chosen, according to a probability p_{sel}, in order to substitute the solution which has been placed on it. This first phase is identical to Poloni-Pediroda *Local Pareto Selection* and it allows to preserve diversity of alleles and genotypes, also increasing speed and robustness of genetic search. In a second phase, the *Shared Fitness Tournament Selection* is carried out: the selected individual is the one that shows the minimum value of shared anti-fitness among the solutions taking part to the local tournament. In other words, differently from Poloni-Pediroda approach, our Tournament Selection does not consider the Pareto dominance concept as basis for the comparison, but it selects the actually more deserving solution.

3.5 Reproduction procedure

We apply the *Evolutionary Direction Operator* [Yamamoto and Inoue, 1995] according to the probability p_{dc}, otherwise we always use a traditional one-point Crossover that is followed by Mutation according to the probability p_{mut}. The string mutation ratio r_{mut} gives the chromosome percentage which is perturbed by the Mutation operator. Finally, we apply the following elitist approach: if the best solutions for each objective (the so-called specialists) have been lost during the reproduction, they are included in the subsequent generation.

4. Simulation results

In this section, we provide a comparison of SPLSDCAS and Poloni-Pediroda technique [Quagliarella et al., 1997], and a weighted sum approach, named MSO (Multiple Single-objective Optimization), serves as additional point of reference. Poloni-Pediroda technique differs from SPLSDCAS because it uses the *Pareto Tournament Selection*, according to which the selected individual is the one that locally dominates the solutions taking part to the tournament defined by a local random walk; besides Poloni approach lacks a procedure specifically oriented to preserve diversity in the population. MSO approach instead optimizes a linear combination of the objectives, using weighting coefficients randomly chosen at the beginning of each evolutionary run. The comparison is carried out using the same seed for the random number generator, resulting in the same initial population; each variable uses 15 bit in order to achieve the same resolution. Finally, each approach involved in the comparison calculates altogether the same number of designs, resulting in the same computational cost.

4.1 Beam problem

This problem is used in order to test SPLSDCAS's effectiveness and its applicability to the real design problems. As shown in fig. 3, we refer to a fixed beam in steel, supported in the middle and loaded by 5 kN on the free extremity, with a length equal to 1000 mm and a thickness equal to 20 mm. The beam horizontal outline is a spline curve with 3 control points so that the only design variables are the beam's heights in the fixed end (H1), in the middle (H2) and in the loaded extremity (H3).

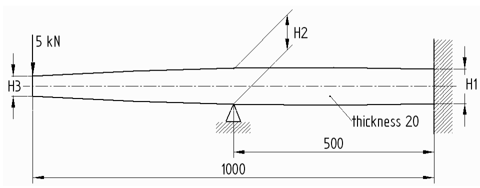

Figure 3. Frontal view of the beam under consideration

We want to minimize the beam's volume and the displacement of the loaded extremity, respecting the axial stress constraint σ_x<200 MPa with a tolerance equal to 10 MPa. The attributes are calculated for each solution through FEM analysis. SPLSDCAS, as shown in fig. 4, overcomes the performances of the other algorithms under consideration because it guarantees the more uniform sampling of the trade-off surface and it provides a Pareto Frontier which dominates most of the Pareto Frontier which has been obtained through the other approaches under consideration.

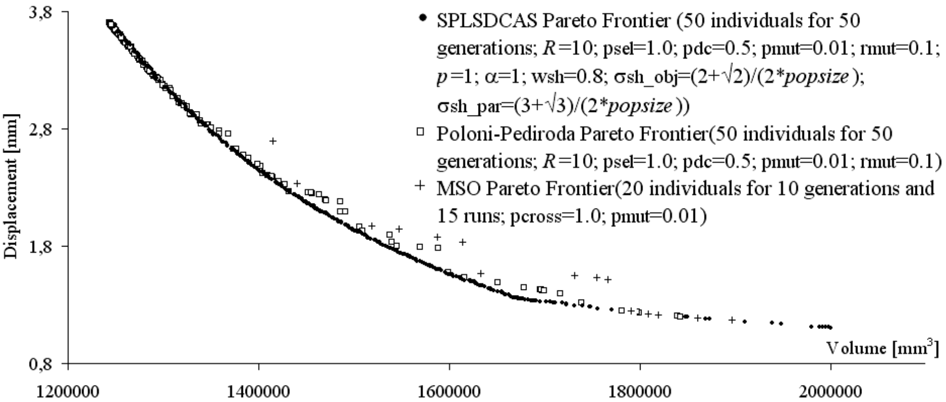

Figure 4. Pareto Frontier relevant to the beam problem

5. Multiobjective shape optimization of a lenticular wheel

Cappello [Cappello et al., 2003] performed the topological optimization of a lenticular bike-wheel, showing that a topology with five spokes is able to minimize the strain energy. In this study, we carry out the multiobjective shape optimization of a lenticular bike-wheel with five spokes, using the SPLSDCAS approach. As shown in fig. 5, we assume that the semi-contour of each spoke is a spline curve passing through 18 control points, which come radially from the origin *Pc*. The design variables are the abscissas of these 18 control points, which have an angular spacing equal to 9° where the contour bend radius is foreseeably small, otherwise this angular spacing is assumed equal to 18°.

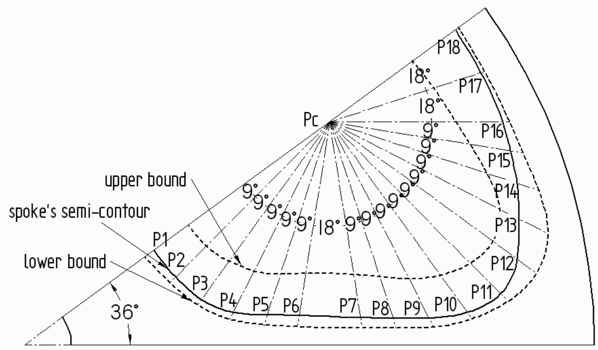

Figure 5. Parametric model of a wheel tenth

We want to minimize simultaneously the wheel mass and the maximum radial displacement of the wheel rim, satisfying the constraints applied to the first principal stress (σ_1<1500 MPa) and to the ovaling, which is defined as the ratio between the radial displacement of the contact point *C* between wheel and ground when *C* belongs to the axis of a spoke and the radial displacement of *C* when it belongs to the middle axis between two consecutive spokes, (ovaling>65%). The attributes are calculated for each solution through FEM analysis. The Pareto Frontier obtained after a computation week, using SPLSDCAS on a CPU at 2.8 GHz, is reported in fig. 6. For the solution 1, which minimizes only the mass, the wheel weighs 0.333 kg and it has a maximum radial displacement u_{max}=2.660 mm. The trade-off solution 2, which shows a maximum radial displacement u_{max}=1.510 mm, gives a weight equal only to 0.383 kg.

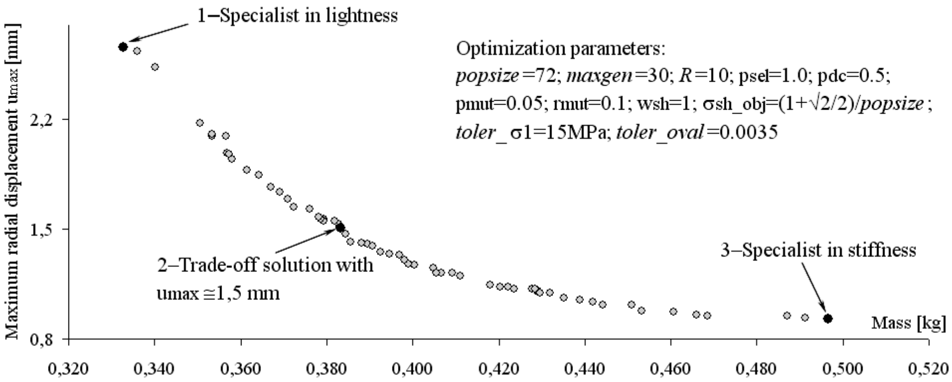

Figure 6. Pareto Frontier and optimization parameters used

The solution 3, which minimizes only u_{max}, furnishes a weight equal to 0.497 kg but as compensation u_{max} is equal to 0.925 mm.

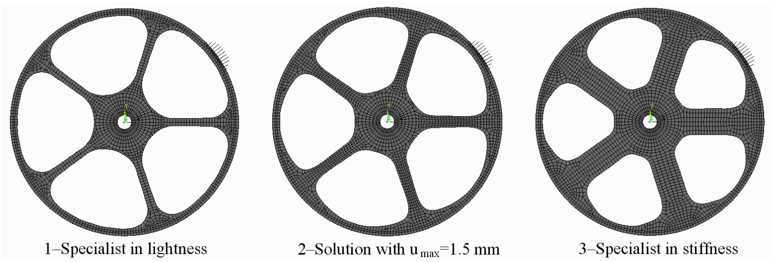

Figure 7. The most significant solutions of the shape optimization problem of a lenticular wheel

6. Conclusions

In this paper, we have proposed a new approach to multiobjective optimisation which is able to outperform the traditional techniques involved in the comparison. Besides, we have proved its effectiveness in handling those design problems with multiple constraints, which show multiple and multi-modal objective functions. - **Fifth world congress of structural and multidisciplinary optimisation, Lido di Iesolo, May 19-23, 2003.**

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Prof. F. Cappello
University of Palermo, Department of Mechanics
Viale delle Scienze, Palermo, Italy, Telephone: +390916657139
E-mail: cappello@dima.unipa.it